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Exploding Bubbles in a Macroeconomic Model*

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ABSTRACT

I describe how equilibrium bubbles emerge in a simplified version of Kiyotaki-Moore (2008). I demonstrate via a numerical example that exploding bubbles may have dramatic and persistent distributional and aggregate effects. I discuss appropriate policy interventions in the wake of a bubble collapse.

*First version: October 23, 2008. This note is a discussion of Kiyotaki-Moore (2008), prepared for the 10/31/08-11/01/08 "Monetary Policy and Financial Frictions" conference at the Federal Reserve Bank of Minneapolis. However, it is written so as to be entirely self-contained. I thank Costas Azariadis for two useful conversations about bubbles in limited enforcement settings. I especially thank Barbara McCutcheon for many interesting conversations about bubbles. Caveat lector: this was written rapidly and is subject to residual error. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Kiyotaki and Moore (2008) (KM) set up a model with entrepreneurs and workers. Workers consume their incomes in each period and have essentially no access to asset markets. Entrepreneurs use accumulated capital and labor to produce output, and create capital goods from consumption goods. A critical feature of the model is that in each period, only a fraction of randomly determined entrepreneurs have this ability to transform consumption into capital. Entrepreneurs are borrowing-constrained, because they are not able to issue equity against the full value of their produced output. As well, and more unusually, entrepreneurs face restrictions on their ability to sell off their holdings of outside equity in others' technologies. If these limits are sufficiently tight, KM show that a zero-dividend asset (that they term money) will be held in equilibrium, even though it is dominated in return by outside equity. The authors study the impact of shocks to productivity and to liquidity on this economy, and discuss possible policy responses to these shocks.

From a technical point of view, this model closely resembles Angeletos (2007)'s model of public and private equity. In both papers, entrepreneurs face idiosyncratic uninsurable risk. This risk affects their project returns (not their labor income). This feature, combined with the assumption that entrepreneurs have CRRA utility, means that the behavior of equilibrium aggregates and prices is independent of the initial distribution of wealth and its future evolution. As a result, it is relatively straightforward to compute equilibrium objects even though markets are incomplete and the distribution of wealth evolves in a nontrivial fashion.

These models are wonderful tools to analyze important economic questions. (For example, Kartashova (2008) uses a calibrated version of Angeletos' framework to assess the economic impact of improving public equity markets.) In this note, I use a model motivated by KM and Angeletos to answer some important topical questions about *bubbles.* I show that in this model, bubbles emerge naturally in the prices of assets that serve as collateral. I then use the model to study the macroeconomic consequences of a bubble's bursting. They can be quite dire. However, despite these potential adverse consequences, bubbles are ex-ante beneficial. Finally, building on recent work by Caballero and Krishnamurthy (2006), I discuss possible government responses to a burst bubble.

In my model, a fraction of entrepreneurs have desirable investment opportunities, while others do not. The arrival of the desirable projects is i.i.d. over entrepreneurs and over time. All entrepreneurs are also endowed with a house. Somewhat extremely, I will assume that this house provides zero consumption services (I conjecture that the analysis goes through if housing services are positive but declining geometrically over time.) A lender can seize a borrower's house, but no other borrower resources. Hence, a borrower's repayment is bounded from above by the value of the house.

Not surprisingly, since houses don't pay dividends, there is a steady-state equilibrium in which houses have zero value. However, there is also another steady-state equilibrium in which the house price is constant and positive. I interpret this positive price as being a bubble in housing prices. I use these equilibria to construct another one with a stochastic bubble. In this equilibrium, there is some small probability of the bubble's bursting at each date. Before the bubble bursts, the house price is positive and constant. After the bubble bursts, the house price reverts to zero forever.

The intuition behind the existence of the bubble is simple, robust, but often ignored. The positive bubble in the housing price exists only because entrepreneurs face occasionally binding borrowing constraints (see Kocherlakota (1992)). The presence of the bubble relaxes this constraint. It allows entrepreneurs to finance more investment, which leads to more production, output, and consumption for everyone in the economy. Bubbles exist only because they serve a useful social purpose.

It is true that (in the stochastic equilibrium) the bubble does eventually collapse. Immediately after this collapse, the entrepreneurs with good projects have little capital available for investment. Macroeconomic aggregates fall dramatically. (This fall takes place in one period; I believe that it can be dragged out over time if investment opportunities were more persistent.) Entrepreneurs then realize that they have to self-finance their projects, and begin to accumulate capital for this purpose. (In any given date, much of the accumulated capital is used inefficiently because the owners do not have useful projects.) The economy transits slowly to a new steady-state level, which is lower than the prior positive-bubble steady state. Entrepreneurs and workers alike mourn the collapse of the bubble. Nonetheless, from an ex-ante perspective, a positive stochastic bubble expands the social pie.

I discuss a range of possible interventions in the wake of the bubble collapse. Following Caballero and Krishnamurthy (2006), I argue that desirable interventions have to accomplish two objectives. First, entrepreneurs have lost their collateral and so can no longer borrow and lend. The government must provide them with a credible way to do so. Second, the government must replace the lost entrepreneurial wealth. There are a number of ways to accomplish these two objectives (although I show that not all current interventions are well-designed along these lines).

My preferred post-collapse intervention is to raise interest rates (that is, lower government debt prices) and thereby sell a large amount of government debt. The government can then apply a fiscal stimulus by distributing the proceeds of this sale among entrepreneurs.

This policy provides a credible vehicle for intertemporal trade (government debt) and provides needed wealth to the entrepreneurs. I show that it completely eliminates the adverse impact of the bubble's collapse on aggregate outcomes.

1. Model Economy

I consider an infinite-horizon economy with a unit measure of entrepreneurs and a unit measure of workers. Workers play little role in this analysis, except to soak up the returns to labor. More specially, each worker supplies one unit of labor inelastically at each date. The workers simply consume their labor income at every date; they do not borrow or lend.

Entrepreneurs maximize the expectation of:

$$(1) \qquad \sum_{t=1}^{\infty} \beta^{t-1} \ln(c_t), 0 < \beta < 1$$

where c_t is consumption at date t. Each entrepreneur has a technology that converts k_{t+1} units of capital installed at date t and n_{t+1} units of labor hired at date $t+1$ into y_{t+1} units of output, according to the production function:

$$(2) \qquad y_{t+1} = A_{t+1} k_{t+1}^{\alpha} n_{t+1}^{1-\alpha}$$

Here, total factor productivity A_{t+1} is a random variable that is i.i.d. over both entrepreneurs and over time. It equals 1 with probability π and 0 with probability $(1-\pi)$. A given entrepreneur learns the value of A_{t+1} at date t (at the time that capital for next period is installed). Capital depreciates at rate δ per period, regardless of the value of A.

Each entrepreneur is endowed with a house. He can buy and sell houses, and houses are infinitely divisible. The entrepreneur can borrow or save using one-period risk-free bonds. In borrowing, his house is the only form of collateral. Hence, the entrepreneur's repayment in period $(t+1)$ is bounded from above by the value of his housing in period $(t+1)$.

Suppose that the entrepreneur faces a house price sequence $(p_t)_{t=1}^{\infty}$, an interest rate sequence $(r_t)_{t=1}^{\infty}$, and a wage sequence $(w_t)_{t=1}^{\infty}$. Then entrepreneur's budget set consists of (c, h, b, k, n) that satisfy:

$$
\begin{aligned}
& c_t(A^{t+1}) + p_t h_{t+1}(A^{t+1}) + b_{t+1}(A^{t+1}) + k_{t+1}(A^{t+1}) \\
\leq \; & b_t(A^t)(1+r_t) + A_t k_t(A^t)^{\alpha} n_t(A^{t+1})^{1-\alpha} + (1-\delta)k_t(A^t) - w_t n_t(A^{t+1}) + p_t h_t(A^t) \text{ for all } t, A^t
\end{aligned}
$$

$$b_{t+1}(A^{t+1})(1+r_{t+1}) \geq -p_{t+1}h_{t+1}(A^{t+1}) \text{ for all } t, A^t$$

$$c_t(A^{t+1}), k_{t+1}(A^t), h_t(A^t) \geq 0 \text{ for all } t, A^t$$

$$b_1 = 0$$

$$h_1 = 1$$

A specification of prices (p, w, r) and entrepreneurial quantities (c, h, b, k, n) form an equilibrium if (c, h, b, k, n) maximizes the entrepreneur's utility among all allocations in his budget set, and markets clear:

$$\sum_{A^{t+1}} \Pr(A^{t+1}) n_t(A^{t+1}) = 1 \tag{3}$$

$$
\begin{aligned}
& \sum_{A^{t+1}} \Pr(A^{t+1}) c_t(A^{t+1}) + \sum_{A^{t+1}} \Pr(A^{t+1}) k_{t+1}(A^{t+1}) \\
= \; & (1-\delta)\sum_{A^t} \Pr(A^t) k_t(A^t) + \sum_{A^{t+1}} \Pr(A^{t+1}) A_t k_t(A^t)^{\alpha} n_t(A^{t+1})^{1-\alpha}
\end{aligned} \tag{4}
$$

$$\sum_{A^{t+1}} \Pr(A^{t+1}) h_{t+1}(A^{t+1}) = 1 \tag{5}$$

$$\sum_{A^{t+1}} \Pr(A^{t+1}) b_{t+1}(A^{t+1}) = 0 \tag{6}$$

2. Two Steady-State Equilibria

In this section, I construct two steady-state equilibria in which all prices and *aggregate* quantities are constant over time. (In both equilibria, the distribution of wealth across entrepreneurs is evolving, even though aggregates are not.) In the first steady-state, housing prices equal zero, and in the second, housing prices are positive.

The construction of these equilibria follows that in KM. The basic idea is that there are two kinds of entrepreneurs at any date t. The first kind knows that his realization of A_{t+1} is 1. His technology has a high return, and so he wants to borrow as much capital as possible to invest in it. Following KM, I'll label these entrepreneurs "investors". The second kind of entrepreneurs knows that his technology has a low return, because $A_{t+1} = 0$. I'll label these entrepreneurs "savers". A critical feature of these models is that entrepreneurs can freely adjust labor demand. Hence, the entrepreneur's payoff from an investment is linear in the amount of capital accumulated. It follows that both kinds of entrepreneurs' decision problems are simply standard portfolio decision problems.

A. No Bubbles

Suppose first that $p_t = 0$ for all t; in such an equilibrium, neither investors nor savers can borrow. Consider an entrepreneur with wealth W_t defined to be:

$$W_t = A_t k_t^{\alpha} n_t^{1-\alpha} + (1-\delta)k_t + b_t$$

(this is the right-hand side of his flow budget constraint in period t). Entrepreneurs consume $c_t = (1-\beta)W_t$ (according to the usual myopic rule associated with log utility), set $k_{t+1} = \beta W_t$ and (without loss of generality) $h_{t+1} = 1$. Investors earn a high gross rate of return on k_{t+1} equal to $(1 - \delta + MPK_{NB})$, where MPK_{NB} is the constant marginal product of capital for investors in this (non-bubble) steady state. Savers earn a low gross rate of return on k_{t+1} equal to $(1-\delta)$. The equilibrium interest rate r^* lies in $(-\infty, -\delta]$ to ensure that no entrepreneur ever wants to buy bonds.

In a steady-state, per-capita[1] wealth must be constant at some level $\overline{W}_{NB}$. Hence:

$$(7) \qquad \overline{W}_{NB} = \pi\beta(MPK_{NB} + 1 - \delta)\overline{W}_{NB} + (1-\pi)\beta(1-\delta)\overline{W}_{NB}$$

This restriction pins down MPK_{NB} to be:

$$(8) \qquad MPK_{NB} = [1-(1-\pi)\beta(1-\delta)]\pi^{-1}\beta^{-1} - 1 + \delta$$

We can then solve for per-capita wealth using the restriction::

$$(9) \qquad MPK_{NB} = \alpha(\beta\overline{W}_{NB})^{\alpha-1}(1/\pi)^{1-\alpha}$$

where we exploit the equilibrium condition that investors hire $1/\pi$ units of labor each.

It is now straightforward to solve for the rest of the equilibrium. Workers earn a constant wage $w_{NB} = (1-\alpha)(\beta\overline{W}_{NB})^{\alpha}(1/\pi)^{-\alpha}$. Entrepreneurs' wealths evolve over time in response to their idiosyncratic shocks, according to the rule:

$$\begin{aligned} (10) \qquad W_{t+1} &= \beta(1-\delta+MPK_{NB})W_t \text{ if } A_{t+1} = 1 \\ (11) \qquad &= \beta(1-\delta)W_t \text{ if } A_t = 0 \end{aligned}$$

Then, at each date, they set k_{t+1} and c_t as above. Note that even though aggregates are constant, the cross-sectional variance of logged entrepreneurial consumption is constantly growing.

[1] Throughout, I use the term "per-capita" to refer to "per-entrepreneur".

B. Constant Bubble

I now construct an equilibrium in which housing prices are constant at a positive level p^*. In this equilibrium, to eliminate arbitrages, housing and bonds are completely equivalent assets. This means that $r_t = 0$ for all t. Without loss of generality, I assume that all entrepreneurs keep their housing levels at $h_t = 1$. In this fashion, the role of housing is serve as collateral.

Now at each date, we define entrepreneurial wealth W_t to be:

(12) $$W_t = A_t k_t^{\alpha} n_t^{1-\alpha} + (1-\delta)k_t + p^* + b_t$$

With this change of adding p^* to wealth, entrepreneurial decision rules are basically the same as above. At date t, investors have an investment opportunity with a gross rate of return equal to $(1-\delta+MPK_{BUB})$, where MPK_{BUB} is the steady-state marginal product of capital. In response, they each borrow as much as possible (set b_t equal to p^*), set $k_{t+1} = \beta W_t$, and set consumption equal to $(1-\beta)W_t$. It is suboptimal for savers to choose a positive level of k_{t+1}, because this plan offers a lower gross rate of return than holding bonds or housing. Instead, savers set bond-holdings b_{t+1} equal to $\beta W_t - p^*$ (because they set $h_{t+1} = 1$) and set consumption equal to $(1-\beta)W_t$. Note that some savers might actually be borrowers (against the values of their houses).

In a steady-state, per-capita wealth must be constant at some level $\overline{W}_{BUB}$. This level satisfies:

(13) $$\overline{W}_{BUB} = \pi\beta(MPK_{BUB} + 1 - \delta)\overline{W}_{BUB} + (1-\pi)\beta\overline{W}_{BUB}$$

which implies that:

(14) $$MPK_{BUB} = [1-(1-\pi)\beta]\pi^{-1}\beta^{-1} - 1 + \delta$$

We can then solve for per-capita wealth as before to satisfy:

(15) $MPK_{BUB} = \alpha(\beta\overline{W}_{BUB})^{\alpha-1}(1/\pi)^{1-\alpha}$

Per capita wealth for savers and investors is the same. Hence, bond market-clearing implies that:

(16) $(1-\pi)(\beta\overline{W}_{BUB} - p^*) - \pi p^* = 0$

so that $p^* = (1-\pi)\beta\overline{W}_{BUB}$.

We can now readily solve for the rest of the equilibrium. Workers earn a constant wage $w_{BUB} = (1-\alpha)(\beta\overline{W}_{BUB})^{\alpha}(1/\pi)^{-\alpha}$. Entrepreneurs' wealths evolve over time in response to their idiosyncratic shocks, according to the rule:

(17)
$$\begin{aligned} W_{t+1} &= \beta(1-\delta+MPK_{BUB})W_t \text{ if } A_{t+1}=1 \\ &= \beta W_t \text{ if } A_{t+1}=0 \end{aligned}$$

They then set c_t, k_{t+1}, b_{t+1} and h_{t+1} as described earlier.

C. Discussion

The behavior of aggregates are determined by the steady-state levels of wealth. It is easy to see that:

(18) $MPK_{NB} > MPK_{BUB}$

which implies in turn that:

(19) $\overline{W}_{BUB} > \overline{W}_{NB}$

There is more wealth with bubbles, which means that per-capita consumption, output, and wages are all higher in the bubble steady-state.

It is true that investors receive a lower return in the bubble steady-state, because the marginal product of capital is lower. However, it is simple to exploit the concavity of the log function to show that:

$$\begin{aligned} (20) \qquad & \pi \ln(1-\delta+MPK_{BUB}) \\ > \ & \pi \ln(1-\delta+MPK_{NB}) + (1-\pi)\ln(1-\delta) \end{aligned}$$

It follows that an entrepreneur, with a given level of wealth, would be happier in the bubble steady-state.

Of course, these are two steady-state equilibria, with different initial levels of capital. It is useful to understand to what extent these differences can be attributed to these different levels of capital. Toward that end, note that in the non-bubble steady state, all entrepreneurs hold capital and so per-capita capital is equal to $\overline{k}_{NB} = \beta\overline{W}_{NB}$. We can write $\overline{k}_{NB}$ in terms of primitives as:

$$\begin{aligned} (21) \quad \overline{k}_{NB} &= \frac{[MPK_{NB}/\alpha]^{1/(\alpha-1)}}{\pi} \\ &= \frac{\{[1-(1-\pi)\beta(1-\delta)]\pi^{-1}\beta^{-1} - 1 + \delta\}^{1/(\alpha-1)}}{\alpha^{1/(\alpha-1)}\pi} \end{aligned}$$

In contrast, in the bubble steady-state, only investors hold capital and so per-capita capital is equal to $\overline{k}_{BUB} = \beta\pi\overline{W}_{BUB}$. We can write $\overline{k}_{BUB}$ in terms of primitives as:

$$(22) \quad \overline{k}_{BUB} = \frac{\{[1-(1-\pi)\beta]\pi^{-1}\beta^{-1} - 1 + \delta\}^{1/(\alpha-1)}}{\alpha^{1/(\alpha-1)}}$$

Remarkably, these formulae imply that steady-state capital is actually *higher* in the non-bubble steady-state.[2]. This result underscores the productive efficiency role of the housing bubble in this economy. Without a bubble, entrepreneurs are forced to self-finance. In many periods, they are unable to exploit their capital effectively because they don't have a good project. They end up accumulating a lot of (wasted) capital just to take advantage of those dates in which their project is actually operational. With a housing price bubble, investors can borrow, and so resources flow readily from savers to investors. There is no need for entrepreneurs to accumulate as much capital.

3. Stochastic Bubbles

I now consider the behavior of the economy in an equilibrium in which housing prices follow a stochastic bubble of the following kind. At each date, the coin is flipped which has a probability ε of coming up heads. If the outcome of the coin flip is tails, the housing price equals a positive number p_{bub}; if the outcome is heads, then the housing price equals zero at date and thereafter.

A. Verbal Description of the Equilibrium

I will treat the case in which ε is near zero, so that the ex-ante probability of the bubble's bursting is small, and assume that the initial per-capita level of capital is approximately $\overline{k}_{bub}$. Then, equilibrium quantities and prices *before* the bubble bursts are well-approximated

[2]It is easy to see that $\overline{k}_{BUB} \leq \overline{k}_{NB}$ if and only if:

$$\pi^{\alpha-1} \geq \frac{\{[1-(1-\pi)\beta(1-\delta)]\pi^{-1}\beta^{-1}-1+\delta\}}{\{[1-(1-\pi)\beta]\pi^{-1}\beta^{-1}-1+\delta\}}$$

The right hand side is increasing in δ and the left-hand side is increasing in α. But this inequality is valid even if $\alpha = 0$ and $\delta = 1$.

by the above bubble steady-state. The only difference that is worth noting is that savers now do hold a positive level of capital to guard against the bubble's bursting. (Some simple algebra shows that savers put a fraction $\beta\varepsilon W_t/\delta$ into capital and $\beta(1-\varepsilon/\delta)W_t$ into savings and housing.)

Now suppose the bubble bursts at date t. The immediate impact is that there is a large and (nearly) unanticipated redistribution of wealth. Houses are now worth zero. Moreover, all of the loans backed by houses will not be repaid. These two shocks cancel each other for the investors (their net investment in housing was zero). But the savers lose all of their wealth, except for the small amount that they had invested in capital to guard against this low-probability event.

Despite this massive redistribution, there is no immediate impact on aggregates. The investors at date t take their accumulated capital, hire workers, and produce output. Wages in period t are unaffected by the bubble's bursting, because they are fully pinned down by the fixed quantities of capital and labor.

Thereafter, the situation changes dramatically. The economy's dynamics are determined by the evolution of per-capita wealth, which is governed by the (nonlinear) difference equations:

$$\begin{aligned}
(23)\qquad \overline{W}_{t+s} &= \beta\pi(1+MPK_{t+s}-\delta)\overline{W}_{t+s-1}+\beta(1-\pi)(1-\delta)\overline{W}_{t+s-1}, s \geq 1 \\
(24)\qquad MPK_{t+s} &= \alpha(\beta\pi\overline{W}_{t+s})^{\alpha-1} \\
(25)\qquad \overline{W}_t &= \beta\pi\overline{W}_{bub}
\end{aligned}$$

(Here, I'm assuming that ε is small, so that I can ignore the capital accumulated by the savers before the bubble's bursting.) Note that, after the bubble collapses, per-capita wealth falls to equal per-capita capital (because all housing wealth is destroyed). Once we know the per-capita level of wealth at each date, it is straightforward to compute aggregate variables in each period. Their time paths are given by:

$$
\begin{aligned}
\overline{c}_{t+s} &= (1-\beta)\overline{W}_{t+s} \\
w_{t+s} &= (1-\alpha)(\beta\pi\overline{W}_{t+s})^{-\alpha} \\
\overline{y}_{t+s} &= (\beta\pi\overline{W}_{t+s})^{\alpha}
\end{aligned}
$$

(Here, $\overline{c}_{t+s}$ represents per-capita consumption.) All of these variables fall sharply from period t to period $(t+1)$ as a result of the bubble collapse, and then transit to a new, lower, steady-state level.

It is important to note that bond interest rates jump downward after the bubble collapses. During the bubble, real interest rates are zero. After the bubble collapses, real interest rates must fall permanently below $-\delta$. This change in interest rates is not driven by changes in risk premia. Instead, after the bubble collapses, entrepreneurial borrowing capacity disappears and the demand for loans declines. The only way to maintain equilibrium in the bond market is if interest rates fall precipitously.

Intermediation is direct in this model. Suppose instead that savers lent and investors borrowed from a common, zero-profit, intermediary. In a bubble steady state, the investors each owe p^* to the intermediaries at each date in the form of debt backed by their houses. After the bubble collapses, the investors will give their (worthless) houses to the intermediaries. The intermediaries are now insolvent: they owe $p^*\pi$ to savers and have no resources with which to make this repayment.

B. Numerical Simulation and Graphical Depiction

In this subsection, I numerically simulate the results of a bubble collapse in this model. The calibration is far from serious (as is necessarily true of any model in which housing is the only form of collateral and in which housing services are zero). However, I believe that the results are intriguing.

I set the parameters as follows:

$$\begin{aligned} \beta &= 0.95, \alpha = 1/3, \delta = 0.1 \\ \pi &= 0.2 \end{aligned}$$

The first three parameters are standard, given that a period length is a year. I picked the last parameter, which is the fraction of entrepreneurs with good projects in a given year, somewhat arbitrarily. (It is sufficiently low to deliver striking results.). I then solve for the path of per-capita output (wages and per-capita consumption behave similarly).

The computed path is depicted in Figure 1. In this figure, output falls by 38% in the first year after the bubble bursts. It grows slowly thereafter. After a decade(!), it is still 35% lower than output before the bubble burst. Eventually (after nearly two decades), output is close to its new steady-state level, 31% below the initial level of output. The simulated path is similar in some respects to the path of output in the United States during the Great Depression.[3]

[3]For this parameterization, the output path displays a form of overshooting as it initially falls below its new long-run level. However, this feature of the output path is not robust. If π is sufficiently close to 1, the post-burst output path may continue to fall after the first period.

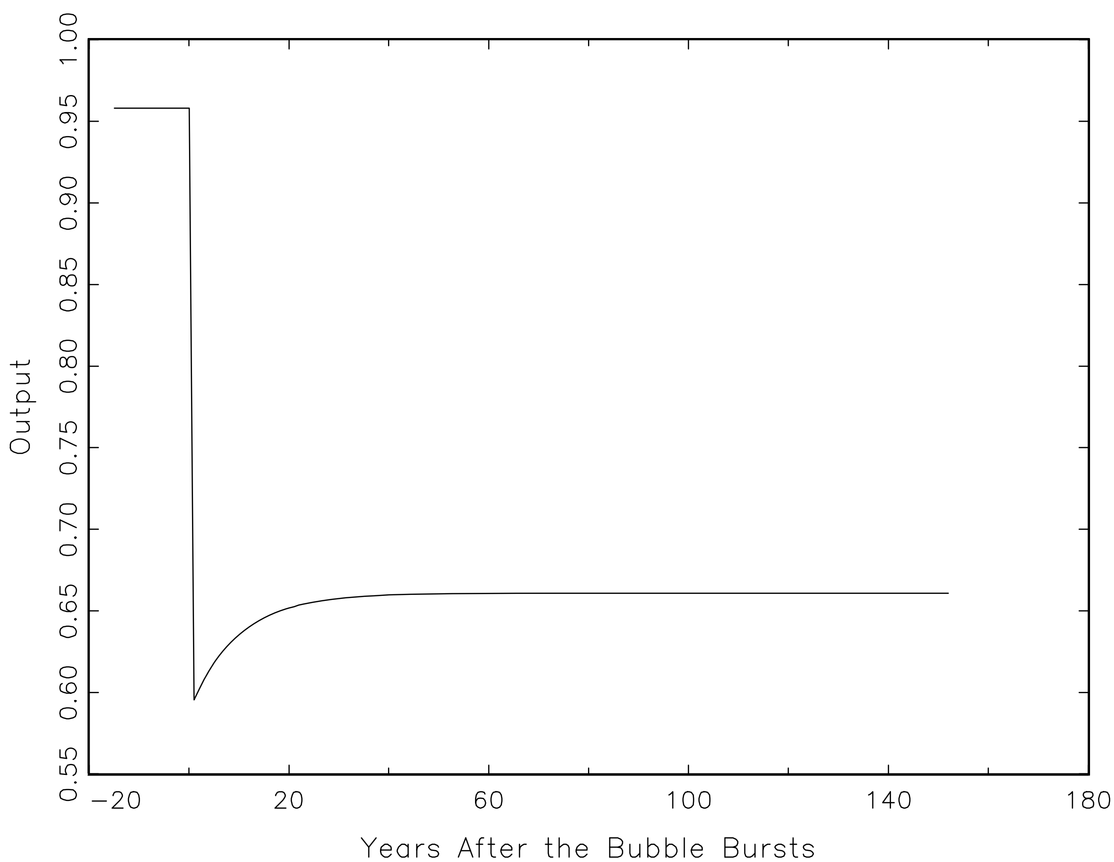

Figure 1: **Output Before and After A Bubble Burst**

The huge fall in output is attributable to the share of wealth that disappears when the bubble collapses. Given these parameter settings, over 75% of per-capita wealth in the initial, bubbly, steady-state is in the form of housing. All of this wealth vanishes in one fell swoop when the bubble collapses. Of course, nothing real disappears. But the ability of the agents in the economy to make use of their resources is greatly impacted - in period $(t+1)$ and forever after.

4. Government Interventions

In this economy, bubble years are good years. The collapse of the bubble triggers a precipitous fall in output which has permanent adverse consequences. Suppose the economy is in the first period after a bubble collapse. What can the government do, if anything, to restore better long-run economic health? I first discuss a class of desirable interventions, and then compare them to what is currently being done in the United States.

A. Backed and Unbacked Debt

In the model economy, the bubble is useful because it creates *collateral*. Once housing prices collapse, there is no source of collateral, and entrepreneurs are forced to self-finance their projects. To help the economy, the government must provide some other source of collateral to the entrepreneurs.

Caballero and Krishnamurthy (2006) (CK) provide useful insights about what these other forms of collateral might be. They analyze an overlapping generations model of an open incomplete markets economy. They use the overlapping generations model to generate bubbles in real estate prices, and study the consequences of these bubbles. My model and theirs differ in important respects, but their discussion of policy is highly relevant.

CK emphasize the role of government debt as an extra source of collateral. They first contemplate unbacked government debt, in which the government re-finances existing debt simply by rolling it over. Such debt is isomorphic to the fiat money in KM and to housing in my model. Suppose that, immediately after the bubble collapses, the government gives each entrepreneur a promise to p^* units of consumption. This promise is unbacked, in the sense that the government will simply default on this promise if it cannot roll it over. There is an equilibrium in which this debt is valued and held. However, there are also other equilibria in which this debt is priced at zero (just like housing is). Once debt is completely unbacked, its ability to operate as collateral is up to the self-fulfilling beliefs of private agents. There is no absolutely no way for the government to guarantee that it will function as a better form of collateral than housing itself is.

CK then consider debt that is explicitly backed by the taxation authority of the government.[4] Suppose that the government hands out claims to future consumption (like TIPS bonds) to entrepreneurs. Then, the entrepreneurs can achieve superior allocations of capital among themselves by trading capital for these real bonds. This bailout policy has almost magical consequences in the model: If the government gives bonds worth p^* to the entrepreneurs when the bubble collapses in period t, aggregates stay at their bubble level forever after. The key point is that this outcome is *guaranteed* to happen, because these *backed* bonds are now fundamentally different from zero-dividend housing. CK argue that in equilibrium, the government may not need to collect the taxes; it may be able to refinance its debt simply

[4]The fundamental premise here is that the government is a superior collection agency than are private agents. From a strict theoretical point of view, this premise is hard to defend (why not just use those great collection powers on behalf of private lenders?). But in reality, it is true that people who don't pay their taxes can go to jail, while debtors cannot. The government seems to have collection powers that it is unwilling to let private creditors use.

through rollover. Nonetheless, its ability to collect the taxes is necessary to its being able to rule out collapses in the value of this debt. Any limitations on this ability to collect taxes (say, because of distortions) curtail the effectiveness of this proposed policy.

To sum up: After a bubble collapses, entrepreneurs have lost borrowing capacity. In the model, the government can completely restore this borrowing capacity by handing out enough backed debt (worth p^*) to entrepreneurs. Within one period, aggregates will jump upward to their level in the bubble steady-state.

B. ***Raise*** Interest Rates and Apply a Fiscal Stimulus

In the above policy, the government simply hands out debt to the entrepreneurs. Alternatively, the government can *sell* the debt to the entrepreneurs. More specifically, suppose that after the bubble collapses, the government offers to sell next period consumption one for one for current consumption. It then applies a fiscal stimulus by distributing the proceeds of this sale evenly among the entrepreneurs.

The government debt offers an interest rate of zero, and so it will be purchased by the savers but not by the investors. Hence, this policy will succeed in selling d units of debt, where d solves:

$$d = \beta(\overline{W}_{bub} - p^* + d)(1 - \pi)$$

This equation uses the fact that savers save a fraction β of their wealth, and their wealth includes the transfer from the government. Recall that $p^* = \beta(1 - \pi)\overline{W}_{bub.}$. We can readily solve this linear equation for d to get:

$$d = \frac{\beta(1 - \pi)(\overline{W}_{bub} - p^*)}{1 - \beta(1 - \pi)}$$

$$
\begin{aligned}
&= \frac{\beta(1-\pi)\overline{W}_{bub}(1-\beta(1-\pi))}{1-\beta(1-\pi)} \\
&= \beta(1-\pi)\overline{W}_{bub} \\
&= p^*
\end{aligned}
$$

Thus, the government's policy raises p^* units of consumption. It distributes this amount to all entrepreneurs, including investors. The investors now have $\overline{W}_{bub}$ available for investing - just as in the bubble steady state. In this fashion, the government has successfully intermediated between the savers and the investors. If the policy is applied sufficiently rapidly, the bubble's collapse may have no impact on aggregate output.

In succeeding periods, the government can repay its outstanding debt by rolling it over. The government's policy re-creates the bubble steady-state. The essence of this policy is that the government offers a *high* interest rate of 0 relative to the prevailing one (less than $-\delta$), and makes a lump-sum transfer to entrepreneurs.

This policy relies on the government's being able to target its fiscal stimulus to entrepreneurs. It may be more realistic to assume that the government must distribute the proceeds of debt sales evenly across all agents, including workers. Using the same parameterization as in Section 3B, I depict the output path that results from this intervention in Figure 2. By comparing this Figure to the earlier Figure 1, we see that the policy dampens the impact of the bubble collapse (in the first period, output falls by 27% not 38%). In the long run, output returns to its initial bubble steady-state, as the amount of government debt in the hands of the entrepreneurs grows to p^*. However, the transition is long.[5] After *twenty*

[5] The policy does generate more consumption for workers at every date after the bubble collapses.

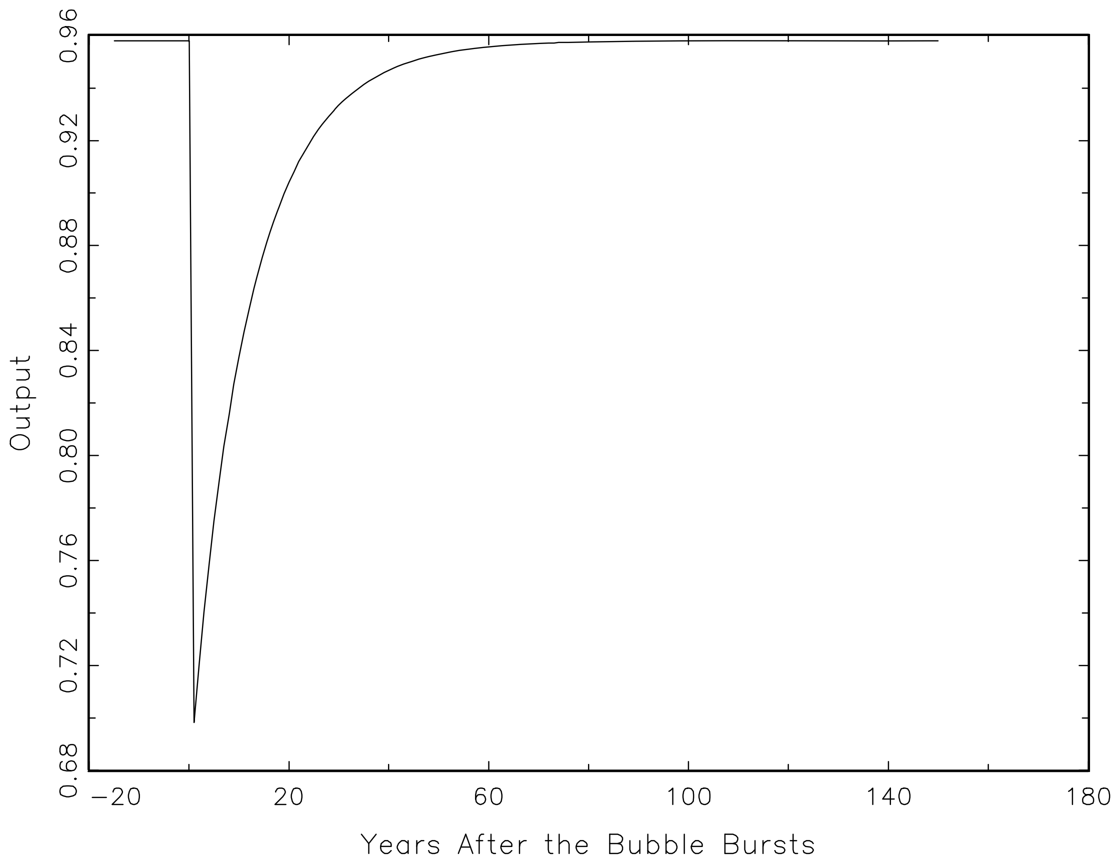

Figure 2: Output Path Resulting From An Untargeted Fiscal Stimulus

years, output is still 6% below the long-run steady-state. The problem with this policy is that not enough wealth gets into the hands of the entrepreneurs.

C. Current Interventions

The federal government is currently intervening in a massive way in financial markets. The model is clearly overly simplistic in a number of important ways. Nonetheless, it is useful to think through the effects of the current federal interventions in the context of the model.

One current government policy is to hand out backed debt to banks. The model does not speak directly to the efficacy of this policy. However, if entrepreneurs cannot credibly commit to repaying their loans to banks, then giving backed debt to banks is useless. The policy needs to get extra wealth into the hands of entrepreneurs with desirable projects if it is to be effective.

The government has also adopted a policy by which it will purchase *unsecured* commercial paper. In the context of the model, suppose the government sells backed debt worth p^* to each entrepreneur in exchange for that entrepreneur's promise to repay p^* next period. This policy has no intrinsic impact on housing prices and so housing prices may well remain zero. In this equilibrium, there is no viable collateral, and the government will get *nothing* back for its loan. Nonetheless the policy has good effects. The entrepreneurs now each have p^* extra units of bonds. Investors can use these bonds to buy capital from the savers. Aggregate activity returns to its bubble steady-state level.

The Federal Reserve is currently *lowering* interest rates. This policy does not work well in the model economy. Lowering interest rates increases the demand for loans. But all of the potential borrowers are already on their borrowing constraint. The correct solution is to *raise* interest rates and redistribute the proceeds of the resulting debt sales. In this fashion, the savers are encouraged to lend to the government, and the government can serve as an intermediary between savers and investors.[6]

Finally, it has been proposed that the government should be willing to buy up houses at pre-collapse prices. We can implement this proposal in the model by the government's offering to buy any house at price p^* (again in exchange for backed government debt). Then, there are two possible kinds of equilibria. In one, some home-owners continue to own some

[6] This policy seems counter-intuitive to those used to working with representative agent models. However, models with liquidity constraints typically imply that central banks should raise interest rates whenever those liquidity constraints get tighter (see Kocherlakota (2003) and KM)..

of their houses. In this kind of equilibrium the price of houses jumps to p^*, and houses are good collateral again. In the other kind of equilibrium, the price of houses is below p^*. Then, the government buys up all houses, but does successfully inject p^* units of backed bonds into the hands of entrepreneurs. Either equilibrium is a good one.

The collapse of a bubble robs entrepreneurs of a way to transfer wealth over time. But the bubble's collapse also robs entrepreneurs of wealth itself. Any successful government interventions must replace both. In the context of the model, there are a number of ways for the government to accomplish these objectives. One method that does not seem to work is to give government debt to financial intermediaries.

5. Conclusions

In this note, I show how bubbles emerge in a simplified version of KM's model. The key to the analysis is that, as in KM's framework, collateral is scarce and all entrepreneurs face borrowing constraints that bind infinitely often into the future. These two ingredients imply that equilibrium bubbles naturally emerge in the price of the collateral. The resulting bubbles expand entrepreneurial borrowing capacity and generate more output, consumption, and welfare. In this framework, the collapse of a bubble has a dramatic and immediate adverse impact on aggregate variables, which thereafter never fully recover.

The model provides a justification for interventions similar to (but definitely distinct from) those that have been considered and implemented in recent weeks. For example, when I set $\pi = 0.2$, the economy requires a large amount of capital re-allocation on an annual basis. If these re-allocations are disrupted, then aggregate output can be surprisingly severely affected within the course of only one year. In this way, the model provides a *possible* justification for the need for speed apparently perceived by the federal government. On the other hand, the model also makes clear that the details of interventions (like whether to raise or lower interest rates!) matter a great deal.

Other recent papers draw connections between collateral scarcity and the existence of bubbles (CK (2006) and Araujo, et al (2006)). However, to gauge the empirical relevance of these theoretical connections, we need to have good measures of entrepreneurial risk and collateral scarcity. As KM point out, models like theirs (and Angeletos (2007)) are specifically constructed to mimic standard macroeconomic frameworks. For this reason, I believe that it will be relatively easy to augment the model in this note so that it is well-suited for a serious quantitative analysis of bubbles in the macroeconomy.

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