Almost Everything We Say is Wrong

Alvarez, Atkeson, Kehoe

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Introduction

How fluctuations in short term interest rates affect the economy crucial for monetary policy:

- standard answer: affect means of other variables, assume it doesn't affect variances,
- data: the opposite.

This paper: make this argument for exchange rates.

Outline

- Standard Models of Monetary Policy
- The Problem
- Extensions
- Implications
- Concluding Remarks

Standard Models of Monetary Policy

Assume monetary authority controls short term interest rate.

Representative Consumer Models

$$\max \sum_t \sum_{s^t} Pr(s^t) \cdot U\left(c_t(s^t)\right) \text{ s.t.}$$

$$\forall t, s^t : \qquad \frac{1}{1+\mathrm{i}_t} b_{t+1}(s^t) + P_t(s^t) c_t(s^t) \leq P_t(s^t) y_t(s^t) + b_t(s^{t-1})$$

FOC:

$$\frac{1}{1+\mathrm{i}_t} = \sum_{s^{t+1} \geq s^t} \frac{U'\left(c_{t+1}(s^{t+1})\right)}{U'\left(c_t(s^t)\right)} \cdot Pr(s^{t+1}|s^t) \cdot \frac{P_t(s^t)}{P_t(s^{t+1})}$$

Representative Consumer Models cont'd

$$
\begin{aligned}
i_t &:= \log(1+\mathrm{i}_t) \Rightarrow \\
\frac{1}{1+\mathrm{i}_t} &= \exp(-i_t) = \beta E_t\left[\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right] \qquad (1)
\end{aligned}
$$

Assume $\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}$ conditionally log-normal. Then:

- $\log\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}$ conditionally normal with:

$$
\begin{aligned}
\mu : &= E_t\left[\log\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right] \\
\sigma^2 : &= var_t\left[\log\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right]
\end{aligned}
$$

Representative Consumer Models cont'd

By $\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}$ log-normal:

$$E_t\left[\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

Use this and take logs in (1) to get:

$$i_t = -\log\beta + E_t\left[-\log\left(\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right)\right] - \frac{1}{2}var_t\left[\log\left(\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right)\right] \quad (2)$$

Representative Consumer Models cont'd

$$i_t = -E_t\left[\log\left(\frac{U_{c,t+1}}{U_{c,t}}\right) - \log\left(\frac{1}{\pi_{t+1}}\right)\right] - \frac{1}{2}var_t\left[\log\left(\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right)\right] - \log\beta$$

Assume $var_t\left[\log\left(\frac{U_{c,t+1}}{U_{c,t}}\frac{1}{\pi_{t+1}}\right)\right]$ constant:

$$i_t = -E_t\left[\log\left(\frac{U_{c,t+1}}{U_{c,t}}\right)\right] + E_t\left[\log\left(\pi_{t+1}\right)\right] + \text{constant} \qquad (3)$$

Interpretation: real and nominal effect.

More General Models

Definition: stochastic discount factor (pricing kernel) for an asset with price q_t and payoff $r_{t+1}(s^{t+1})$ is a r.v. $m_{t+1}(s^{t+1})$ s.t.:

$$q_t = E_t \left[m_{t+1}(s^{t+1}) r_{t+1}(s^{t+1}) \right]$$

Define asset return $R_{t+1}(s^{t+1}) := \frac{r_{t+1}(s^{t+1})}{q_t}$. Then:

$$1 = E_t \left[m_{t+1}(s^{t+1}) R_{t+1}(s^{t+1}) \right]$$

More General Models cont'd

Then can rewrite (1) - (3) as
(use $\forall s^{t+1} \geq s^t : R_{t+1}(s^{t+1}) = 1 + i_t = \exp(i_t)$):

$$\begin{aligned} \exp(-i_t) &= E_t\left[m_{t+1}\right] && (4) \\ i_t &= E_t\left[-\log\left(m_{t+1}\right)\right] - \frac{1}{2} var_t\left[\log\left(m_{t+1}\right)\right] && (5) \\ i_t &= -E_t\left[\log\left(m_{t+1}\right)\right] + \text{constant} \end{aligned}$$

Adding a Foreign Country

$$
\begin{aligned}
i_t^* &= E_t\left[-\log\left(m_{t+1}^*\right)\right] - \frac{1}{2} var_t\left[\log\left(m_{t+1}^*\right)\right] && (6)\\
i_t - i_t^* &= E_t\left[\log\left(m_{t+1}^*\right) - \log\left(m_{t+1}\right)\right] - p_t && (7)\\
p_t &:= \frac{1}{2}\left[var_t\left[\log\left(m_{t+1}^*\right)\right] - var_t\left[\log\left(m_{t+1}\right)\right]\right] && (8)
\end{aligned}
$$

Note: p_t assumed constant in standard models

The Problem

Data: exchange rates are random walks:

$$\begin{aligned} \log(e_{t+1}) &= \log(e_t) + \varepsilon_{t+1} \\ E_t[\log(e_{t+1})] &= \log(e_t) + \mu_\varepsilon \end{aligned}$$

μ_ε mean of the step distribution, constant over time.

A Contradiction

$$\max \sum_{t} \sum_{s^t} Pr(s^t) \cdot U\left(c_t(s^t)\right) \text{ s.t.}$$

$$\forall t, s^t : \quad \sum_{s^{t+1} \geq s^t} a_{t+1}(s^{t+1}) + \sum_{s^{t+1} \geq s^t} \frac{a^*_{t+1}(s^{t+1})}{e_t(s_t)} + P_t(s^t) c_t(s^t) \leq$$

$$\leq P_t(s^t) y_t(s^t) + R_t(s^t) a_t(s^t) + \frac{R^*_t(s^t) a^*_t(s^t)}{e^t(s^t)}$$

Note: $e = \frac{home \ \ currency}{foreign \ \ currency}$

A Contradiction cont'd

By definition:

$$
\begin{aligned}
m_{t+1}(s^{t+1}): \quad & 1 = E_t\left[m_{t+1}(s^{t+1})R_{t+1}(s^{t+1})\right] \\
m^*_{t+1}(s^{t+1}): \quad & 1 = E_t\left[m^*_{t+1}(s^{t+1})R^*_{t+1}(s^{t+1})\right]
\end{aligned}
$$

No arbitrage:

$$
\begin{aligned}
R_{t+1}(s^{t+1}) &= R^*_{t+1}(s^{t+1})\frac{e_{t+1}(s_{t+1})}{e_t(s_t)} \\
\Longrightarrow m^*_{t+1} &= m_{t+1}\frac{e_{t+1}}{e_t} \qquad (9)
\end{aligned}
$$

Complete markets: unique pricing kernel.

A Contradiction cont'd

Take logs and conditional expectations in (9):

$$E_t\left[\log(e_{t+1})\right] - \log(e_t) = E_t\left[\log(m^*_{t+1})\right] - E_t\left[\log(m_{t+1})\right] \quad (10)$$

Then plug into (7):

$$\begin{aligned} i_t - i^*_t &= E_t\left[\log\left(e_{t+1}\right) - \log\left(e_t\right)\right] - p_t \quad (11) \\ i_t - i^*_t &= \mu_\varepsilon - p_t \end{aligned}$$

Recall:

$$p_t := \frac{1}{2}\left[var_t\left[\log\left(m^*_{t+1}\right)\right] - var_t\left[\log\left(m_{t+1}\right)\right]\right]$$

An Interpretation

p_t is excess return, thus represents risk premium:

- return on home (dollar) bond: $\exp(i_t)$,
- return on foreign bond (uncertain): $\frac{1}{e_t}\exp(i_t^*)e_{t+1}$.

Expected excess log return of foreign bond:

$$\begin{aligned} E_t\left[\log\left(\frac{1}{e_t}\exp(i_t^*)e_{t+1}\right) - \log\left(\exp(i_t)\right)\right] &= \\ E_t\left[-\log(e_t) + i_t^* + \log(e_{t+1}) - i_t\right] &= \\ -\log(e_t) + i_t^* + E_t\left[\log(e_{t+1})\right] - i_t &= p_t \end{aligned} \tag{12}$$

An Interpretation cont'd

- Dollar return of foreign bond uncertain $\Longrightarrow$ excess return compensates the holder of foreign bond for this risk.
- $p_t \geq 0$? Maybe not.

  Suppose $E_t\left[\log(e_{t+1})\right] = e_t$ and $i_t \neq i_t^*$. Then either $p_t > 0, p_t^* < 0$ or $p_t < 0, p_t^* > 0$ since $p_t = i_t^* - i_t = -p_t^*$.
- Alternatively p_t represents differences in liquidity services, transaction costs or tax rates.

An Interpretation cont'd

Main point:

if exchange rates are random walks then changes in interest rate differentials are changes in risk.

Extensions

Incomplete market models $\Longrightarrow$ pricing kernels not unique.

- Brandt, Cochrane, Santa-Clara (2006): $m^*_{t+1} = m_{t+1}\frac{e_{t+1}}{e_t}$ still valid for minimum variance pricing kernels.
- Another example - incomplete markets model; 3 assets: home bond, foreign bond and foreign currency.

Extensions cont'd

$$\max \sum_t \sum_{s^t} Pr(s^t) \cdot U\left(c_t(s^t)\right) \text{ s.t.}$$

$$\forall t, s^t : \quad b_{t+1}(s^t) + \frac{b^*_{t+1}(s^t)}{e_t(s_t)} + P_t(s^t)c_t(s^t) \leq$$

$$\leq P_t(s^t)y_t(s^t) + \exp\left[i_t(s^t)\right] b_t(s^{t-1}) + \frac{\exp\left[i^*_t(s^t)\right] b^*_t(s^{t-1})}{e^t(s^t)}$$

By definition any pricing kernel:

$$\begin{aligned} 1 &= E_t\left[m_{t+1} \exp\left(i_t\right)\right] \\ 1 &= E_t\left[m_{t+1} \exp\left(i^*_t\right) \frac{e_{t+1}}{e_t}\right] \end{aligned}$$

Extensions cont'd

Take logs, assume log-normality of m_{t+1} and $m_{t+1}e_{t+1}$, use variance of a sum:

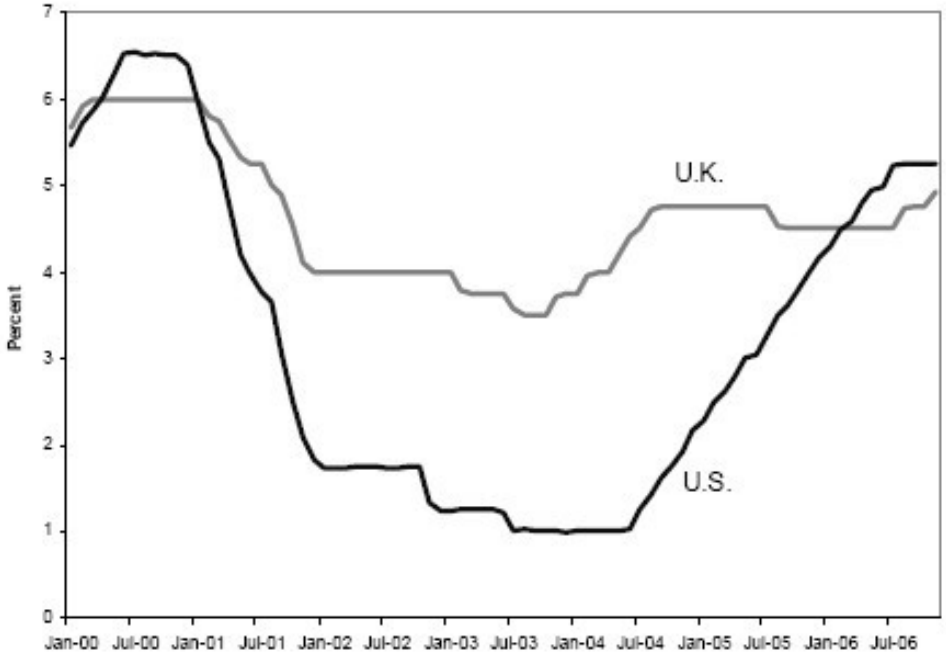
$$\begin{aligned} i_t - i_t^* = \quad & E_t\left[\log\left(e_{t+1}^*\right)\right] - \log\left(e_t\right) + \\ & \frac{1}{2}var_t\left[\log\left(\frac{e_{t+1}}{e_t}\right)\right] + cov_t\left[\log\left(m_{t+1}\right), \log\left(\frac{e_{t+1}}{e_t}\right)\right] = \\ & \mu_\varepsilon + \frac{1}{2}var_t\left[\log\left(\frac{e_{t+1}}{e_t}\right)\right] + cov_t\left[\log\left(m_{t+1}\right), \log\left(\frac{e_{t+1}}{e_t}\right)\right] \end{aligned} \tag{13}$$

Arguing Causality

Change in policy, i.e. $i_t - i_t^* \Longrightarrow$ change in risk premia.

Figure 1A. U.S. Policy Changed More Dramatically Than Britain's

U.S. Federal Funds Rate and U.K. Official Bank Rate
Monthly, January 2000–November 2006



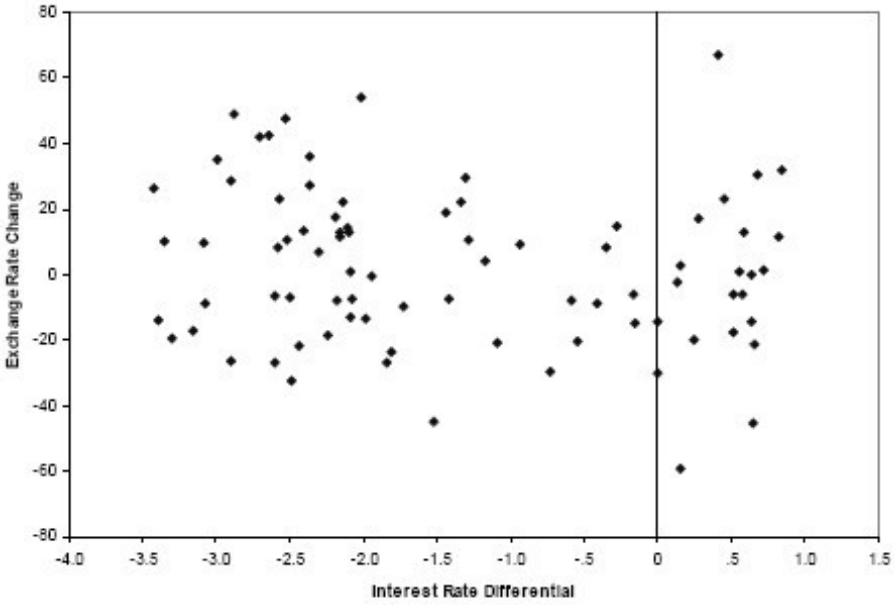
Sources: U.S. Federal Reserve Board of Governors, Bank of England

Arguing Causality cont'd

$i_t - i_t^*$ uncorrelated with $\log(e_{t+1}) - \log(e_t)$.

Figure 1B. The Policy Difference Was Not Related to Changes in Exchange Rates

U.S. and U.K. Interest Rate Differential vs. Exchange Rate Change
Annualized, 2000–2006



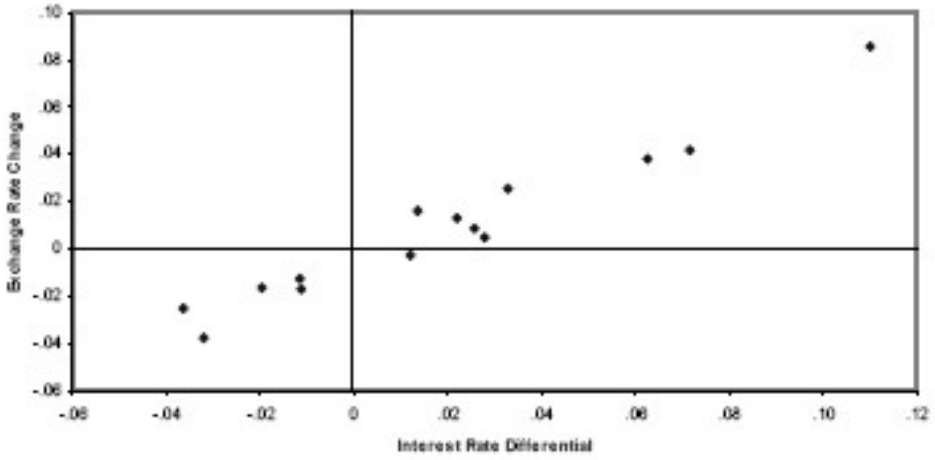
Sources: U.S. Federal Reserve Board of Governors, Bank of England

Using Old Models

Useful for cross sections of long-run averages.

Figure 2. Standard Models Can Account for Some Long-Run Interest Rate Differences Across Countries

Average One-Month Interest Rate Differential (With U.S.) vs.
Average One-Month Change in Exchange Rate (With U.S. Dollar)
January 1976–March 1998, for 14 Countries*



*The countries are Austria, Belgium, Canada, Denmark, England, France, Germany, Italy, Japan, the Netherlands, Portugal, Spain, Sweden, and Switzerland.

Sources: Ravi Bansal and Magnus Dahlquist

Designing New Models

- Develop models in which excess returns fluctuate with differences in liquidity services, transaction costs, tax rates.
- Develop models in which monetary policy affects risk. AAK, 2006 - monetary policy affects market segmentation.

Concluding Remarks

How asset prices (short term interest rate) affect the economy crucial for monetary policy - critical look at existing models.

- Data: interest rate differentials reflected in excess return movements.
- Exchange random walks $\Longrightarrow$ fluctuations of interest rate differentials accounted for by fluctuations in conditional variances rather than conditional means (existing models).
- Applicable to excess returns of long over short term interest bonds.