Monetary Policy in a Channel System

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Channel System

Central Bank offers 2 overnight facilities:

1. lending facility - i_l, collateralized
2. deposit facility - i_d

- pure channel system monetary policy: changes in i_l and/or i_d
- money supply endogenous (no OMO)

Channel System (cont'd)

Stylized facts:

1. positive spread $i_l - i_d > 0$
2. monetary policy: keep spread constant, move corridor
3. money market rate $\sim \frac{i_l + i_d}{2}$

Questions:

1. differences in spreads across countries
2. differences in degree of control over interbank overnight rates

Related Literature

Papers analyzing the channel system:

- Woodford (2000, 2001, 2003)
- Whitesell (2006)
- Gaspar, Quiros, Menindizabal (2004)
- Guthrie, Wright (2000)
- Heller, Lengwiler (2003)

$\Rightarrow$ partial equilibrium, no GE model exists, because in the channel system:

1. money supply endogenous
2. details of monetary policy believed not to matter

Goals of the paper

Build DGE model to study monetary policy in the channel system:

1. optimal interest rate corridor
2. optimal collateral policy
3. corridor changes $\Rightarrow$ money market rate changes

Preview of results:

1. $i_l - i_d \geq 0$ if cost of holding collateral ≥ 0
2. $i_l - i_d$ decreasing in rate of return of collateral
3. equivalent policies: change spread or move the corridor
4. money market $>$ target rate if cost of holding collateral > 0

Monetary Policy in the Eurozone

Operating procedures of the ECB:

- weekly refinancing operations (minimum bid rate + quantity announced, banks bid for liquidity)
- lending rate $i_l = \textit{minimum bid rate} + 1\%$ (either repo or collateralized loans)
- deposit rate $i_d = \textit{minimum bid rate} - 1\%$ $\Rightarrow$ spread 200 basis points

Monetary Policy in the Eurozone (cont'd)

Euro money markets feature:

After the money market "closes", a bank can still access the ECB's standing facilities if:

- receives liquidity, can save at i_d
- needs liquidity, can borrow at i_l

$\Rightarrow$ modelled in the paper

Outline

- Environment
- No Trade in the Money Market
- Trade in the Money Market
- Policy Implications
- Conclusion

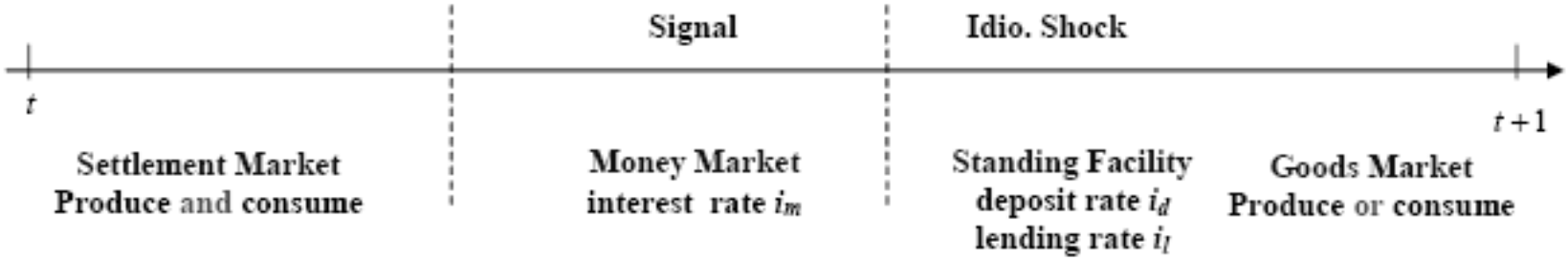
Environment

The Basics:

- measure 1 of agents and a central bank
- discrete time, infinite horizon
- sequence of 3 markets within a period

Environment (cont'd)

Timing of Markets:



Goods Market

- competitive market
- n probability of being buyer, $1 - n$ seller
- utility function:
  $u(q), u'(q) > 0, u''(q) < 0, u'(0) = +\infty, u'(\infty) = 0$
- cost of production: $c(q) = q$
- trades anonymous, trading histories private $\Rightarrow$ money essential
- discount factor $\beta > n$
- price of consumption p

Money Market

Signals:

- 2 possible signals: H, L
- $\mathrm{P}(H) = \sigma_H$, $\mathrm{P}(L) = \sigma_L = 1 - \sigma_H$
- $\mathrm{P}(\text{seller}|k) = n^k, \ k \in \{H, L\}$
- $n = \sum_{k \in \{H,L\}} \sigma^k n^k$
- $\varepsilon := n^H - n^L \in [0, 1]$

Cases: $\varepsilon = 0, \varepsilon = 1, \varepsilon \in (0, 1)$

Then lend/borrow money in a competitive market

Settlement Market

- produce, consume general good, repay loans, redeem deposits, adjust money holdings (m), collateral holdings (b)
- production linear in labor, q = l
- u(c,l) = c - l
- agent never works and consumes at the same time $\Rightarrow$ can denote h labor and $-h$ consumption
- linearity: b, m degenerate (compare to Lagos, Wright)
- price of consumption P

Monetary Policy

Standing Facility:

- at the beginning of period 3 (after uncertainty is realized) can borrow at i_l
- at the end of period 3 (after trades) can deposit at i_d
- $i_l > i_d$

Monetary Policy (cont'd)

Collateral:

- general good can be stored - return $R; R \geq 1, \beta R \leq 1$
- has to be used as collateral on loans from CB and in the money market (CB keeps track of transactions)
- cannot be used in transactions between agents in goods market

necessity for existence of collateral to prevent default (?)

Lump-sum money transfers:
$\pi M \Rightarrow$ LoM for aggregate money:

$$M_{+1} = M - i_l L + i_d D + \pi M$$

Monetary Policy (cont'd)

Definition:

$$\begin{aligned} \text{spread } \delta &:= i_l - i_d \\ \text{target rate } i_p &:= (i_l + i_d)/2 \end{aligned}$$

Monetary policy: change δ or i_p

First Best Allocation

Welfare if in a stationary allocation from 1st period (*including the initial production of b*):

$$(1-\beta)\mathcal{W} = (1-n)[u(q)-q] + (\beta R - 1)b$$

Solution: $q = q^*, b = 0$ unless $\beta R = 1$

Note: $\beta R < 1$ costly collateral

No Trade in the Money Market

- $\varepsilon = 0 \Rightarrow$ signal has no information, agents the same $\Rightarrow$ no trade in money market
- focus on *stationary symmetric equilibria*
  1. $\frac{M_{t+1}}{P_{t+1}} = \frac{M_t}{P_t}$
  2. $\gamma := \frac{M_{t+1}}{M_t} = \frac{P_{t+1}}{P_t}$
- $\phi_t := \frac{1}{P_t} \Rightarrow \gamma = \frac{\phi_t}{\phi_{t+1}}$
- $V(m, b), W(m, b, l, d)$

Settlement Market

$$W(m,b,l,d) = \max_{h,m_2,b_2} -h + V(m_2,b_2)$$

s.t.

$$\begin{aligned} \phi m_2 + b_2 &= h + \phi m + Rb + \phi(1+i_d)d - \phi(1+i_l)l + \phi\pi M \\ b_2 &\geq 0 \\ m_2 &\geq 0 \end{aligned}$$

Settlement Market (cont'd)

FOC: $V_m \leq \phi, V_b \leq 1$

$\Rightarrow m_2, b_2$ independent of state (degenerate)

ET: $W_m = \phi, W_b = R, W_l = -\phi(1 + i_l), W_d = \phi(1 + i_d)$

Goods Market

$$\begin{aligned} V(m,b) &= (1-n)[u(q)+\beta W(m-pq-d_b+l_b,b,l_b,d_b)]+ \\ &\quad +n[-q_s+\beta W(m+pq_s-d_s+l_s,b,l_s,d_s)] \end{aligned}$$

- a seller doesn't take a loan: $l_s = 0$
- a seller deposits everything: $d_s = m + pq_s$
- a buyer borrows just enough: $l_b = pq - m$
- a buyer doesn't make a deposit: $d_b = 0$
  o/w money accumulated - not (stationary) equilibrium

Goods Market (cont'd)

Using FOC and ET from before:

- seller: $1 = p(1 + i_d)\phi_{+1}\beta$
- buyer's collateral constraint: $\{\beta\phi_{+1}\lambda_l\} : l \leq \frac{Rb}{\phi_{+1}(1+i_l)}$

  $\Rightarrow u'(q) = \frac{1+i_l+\lambda_l}{1+i_d} \Rightarrow$ inefficiency unless $\lambda_l = 0, i_l = i_d$

- and also value of money:

  $$V_m = \frac{(1-n)u'(q)}{p} + n\beta\phi_{+1}(1 + i_d)$$

Liquidity Premium

Value of collateral:

$$V_b = \beta R + \frac{(1-n)\lambda_l \beta R}{1+i_l} \geq \beta R$$

Liquidity Premium:

$$1 - \beta R = (1-n)\beta R \left[u'(q) \frac{1+i_d}{1+i_l} - 1 \right] \geq 0$$

Symmetric Stationary Equilibrium

Definition:

$$\Delta = \frac{1+i_l}{1+i_d}$$
$$z_m = \frac{m}{p}$$
$$z_l = \frac{l}{p}$$

Symmetric Stationary Equilibrium (cont'd)

Definition: A symmetric stationary equilibrium is a policy (i_d, i_l, π) and a time invariant list (γ, q, z_l, z_m, b) with $z_l \geq 0, z_m \geq 0$ s.t.

$$\begin{aligned}
\frac{1-\beta R}{\beta R} &\geq (1-n)\left[\frac{u'(q)}{\Delta}-1\right] (= \text{ if } b>0) \\
\frac{\gamma-\beta(1+i_d)}{1+i_d} &\geq (1-n)\left[u'(q)-1\right] (= \text{ if } m>0?) \\
q &= z_m + z_l \\
\gamma &= 1+i_d-(1-n)(i_l-i_d)\frac{z_l}{z_m}+\pi \\
z_l &= \frac{\beta R b}{\Delta} (\text{since } b>0 \Rightarrow \lambda_l > 0)
\end{aligned}$$

Symmetric Stationary Equilibrium (cont'd)

Note: 5 equations, 5 unknowns, the rest can be determined

- z_d, q_s from market clearing
- h_b, h_s from BC, FOC (9) in paper and definition of γ

Symmetric Stationary Equilibrium (cont'd)

Definition: $\tilde{\Delta} := \frac{1-\beta n+\pi/(1+i_d)}{1/R-n\beta}$

Proposition 1: $\forall(i_d, i_l, \pi)$ s.t. $i_l \geq i_d \geq 0, \exists!$ symmetric stationary equilibrium s.t.

- $z_l > 0$ and $z_m = 0 \Leftrightarrow \Delta = 1$
- $z_l > 0$ and $z_m > 0 \Leftrightarrow 1 < \Delta < \tilde{\Delta}$
- $z_l = 0$ and $z_m > 0 \Leftrightarrow \Delta \geq \tilde{\Delta}$

Optimal Policy

Goal: maximize representative agent utility subject to equilibrium conditions.

Implementability constraints:

- If $\Delta \geq \tilde{\Delta}$ (lower bound on q):

$$\tilde{q} = u'^{-1}\left(\frac{1 - \beta n + \pi/(1 + i_d)}{\beta(1 - n)}\right)$$

- Upper bound on q (attained for $\Delta = 1$):

$$\hat{q} = u'^{-1}\left(\frac{1/(\beta R) - n}{1 - n}\right)$$

Optimal Policy (cont'd)

For $1 \leq \Delta \leq \tilde{\Delta}$:

$$\frac{1-\beta R}{\beta R} = (1-n)[u'(q)/\Delta - 1]$$

$$q = \beta R b F(\Delta; \pi),$$

where:

$$F(\Delta; \pi) = \frac{1}{\Delta}\left[1 + \frac{(1-b)(\Delta-1)}{1+\beta n(\Delta-1) - \Delta/R + \pi/(1+i_d)}\right]$$

Optimal Policy (cont'd)

Optimal policy therefore solves:

$$\max_{q,b,\pi}(1-n)[u(q)-q]+(\beta R-1)b$$

$$\text{s.t. } q=\beta RbF\left(\frac{\beta R(1-n)u'(q)}{1-n\beta R};\pi\right), \quad \tilde{q}(\pi)\leq q\leq \hat{q}$$

Proposition 2: $\pi = 0$ is optimal. There exists a critical value $\bar{R}$ such that, if $R < \bar{R}$ then optimal policy is $\Delta \geq \tilde{\Delta}$. Otherwise, the optimal policy is $\Delta \in (1, \tilde{\Delta})$.

Proposition 3: When $R \to 1/\beta$ optimal policy $i_l \to i_d$ implements first-best allocation q^*. Price level approaches infinity.

Trade in the Money Market

Suppose now $\varepsilon > 0$ (signal conveys some information about agents type which will be revealed in the goods market).

- Agents who receive signal H are more likely to become sellers $\Rightarrow$ have incentive to save (lend).
- Agents who receive signal L are more likely to become buyers $\Rightarrow$ have incentive to borrow money.

Settlement market

Agent enters settlement market with m units of money, b collateral, l loans, d deposits and private credit y.
Bellman equation:

$$W(m, b, l, d, y) = \max_{h, m_2, b_2} -h + Z(m_2, b_2)$$

$$\text{s.t. } \phi m_2 + b_2 = h + \phi m + Rb + \phi(1 + i_d)d - \phi(1 + i_l)l - \phi(1 + i_m)y$$

Money Market

Let y^k be amount of money borrowed after signal S_k, $k \in \{H, L\}$.

$$Z(m, b) = \sum_{k \in \{H,L\}} \sigma^k V^k(m + y^k, b, y^k),$$

where y^k solves

$$\max_{y^k} V^k(m + y^k, b, y^k)$$

s.t.

$$\{\phi_{+1}\beta\lambda^k_{ml}\} : y^k \leq Rb/[\phi_{+1}(1 + i_m)],$$
$$\{\phi_{+1}\beta\lambda^k_{md}\} : m + y^k \geq 0.$$

Money Market (cont'd)

FOC:

$$V_m^k + V_y^k - \phi_{+1}\beta\lambda_{ml}^k + \phi_{+1}\beta\lambda_{md}^k = 0$$

Market clearing:

$$\sum_{k\in\{H,L\}} \sigma^k y^k = 0$$

Goods Market

$$V^k(m,b,y) = \max\ (1-n^k)[u(q^k) + \beta W(m - pq^k + l^k, b, l^k, 0, y)] \\ + n^k[-q_s^k + \beta W(m + pq_s^k - d^k, b, 0, d^k, y)]$$

Define $\hat{\Delta} = (1+i_l)/(1+i_m)$. Constraints are:

$$l^k \leq \bar{l}^k := Rb/[\phi_{+1}(1+i_l)] - y^k/\hat{\Delta}$$
$$pq^k \leq m + l^k$$
$$d^k \leq m$$

Money Supply

$$M_{+1} = M - [\sigma^H(1-n^H)l^H + \sigma^L(1-n^L)l^L]i_l + [\sigma^H n^H d^H + \sigma^L n^L d^L]i_d$$

$$\frac{M_{+1}}{M} = 1 + i_d - (i_l - i_d)[\sigma^L(1-n^L)l^L/M + \sigma^H(1-n^H)l^H/M]$$

Symmetric Stationary Equilibrium

Lemma 4: In a symmetric stationary equilibrium with no short selling constraint we have:

$$
\begin{aligned}
R\gamma =& 1 + i_m, \\
\frac{\beta R b}{\Delta} =& \sigma^H q^H + \sigma^L q^L - z_m, \\
z^H =& -\sigma^L(\sigma^L - \sigma^H)\frac{\hat{\Delta}}{\hat{\Delta}-1} = \frac{\sigma^L}{\sigma^H} z^L, \\
z_m =& \frac{\hat{\Delta}}{\hat{\Delta}-1}\frac{(\hat{\Delta}-1)[\sigma^L(1-n^L)q^L + \sigma^H(1-n^H)q^H]R(\Delta-1)}{R\hat{\Delta} - \Delta + (1-n)R\hat{\Delta}(\Delta-1)} \\
& - \frac{\hat{\Delta}}{\hat{\Delta}-1}\frac{\varepsilon\sigma_L\sigma_H(\sigma^L-\sigma^H)\hat{\Delta}R(\Delta-1)}{R\hat{\Delta} - \Delta + (1-n)R\hat{\Delta}(\Delta-1)},
\end{aligned}
$$

Symmetric Stationary Equilibrium (cont'd)

$$\hat{\Delta} = \frac{\Delta}{n\beta R(1-\Delta)+\Delta},$$

$$u'(q^k) = \frac{n^k}{1-n^k}\Delta\frac{1-n\beta R}{n\beta R},$$

$$b \geq 0,$$

$$z^L < \beta R b\hat{\Delta}/\Delta,$$

$$z^H > -z_m.$$

Symmetric Stationary Equilibrium (cont'd)

Proposition 5: For any $1 < \Delta < \tilde{\Delta}$, there exists $\varepsilon_1 > 0$, such that for $\varepsilon < \varepsilon_1$ a symmetric monetary equilibrium exists where no short-selling constraint in the money market binds.

Policy Implications

Recall $i_p = (i_l + i_d)/2$, $\delta = i_l - i_d$.
Money market rate:

$$i_m = i_l - n\beta R\delta$$

Fisher equation:

$$\pi = (1 + i_m)/R - 1$$

$$\pi = (1 + i_l)/R - n\beta\delta - 1$$

Tight policy through increasing spread:

$$\Delta = \frac{1 + i_l}{1 + i_d} = \frac{1 + i_l}{2i_p - i_l}$$

Conclusions

- First DGE model of channel system.
- Money market rate is in corridor set by central bank $\sim (i_d + i_l)/2$.
- Interest rate rules meaningless, i.e. spread is also a policy instrument.

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- Money market rate is in corridor set by central bank $\sim (i_d + i_l)/2$.
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- Open questions / for future research:
  - optimal policy in general framework,
  - volatility of money market rate,
  - aggregate shocks.