Discussion of:
Payment networks in a search model of money

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Motivation

- Take a standard, generation III monetary search model
- Now add a *central database* to the environment which
  - Keeps track of history of agents' trades
  - Allows member agents to trade without money

The questions

In such an environment . . .

- What is the efficient allocation?
- What is the incentive constrained allocation?
- What policies can a planner implement to obtain the efficient allocation?

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1 An environment with a central database

- Time is discrete, $t = 0, 1, \ldots$
- Mass 1 of infinitely-lived agents
- $k \geq 3$ types of agents, randomly matched each period
- $k \geq 3$ types of perishable consumption goods each period

Environment

- A type i agent
  - Consumes good i, gets utility $u(c)$
  - Produces good $i+1$, modulo k, at cost $c > 0$
  - No double coincidence of wants: single coincidence meetings
- **A1:** $\exists\ \hat{c} \in \mathbb{R}$ s.t. $u(\hat{c}) = \hat{c}$ and $u(c) > c$ if $c \in (0, \hat{c})$
- Agents discount future at rate $\beta \in (0, 1)$

Means of exchange: Money

- Mass M of perfectly durable objects called money
- Money
  - Gives no utility to agents
  - Is indivisible
  - Has unit upper bound on holdings

Means of exchange: Central database

- Agents can pay for access to a *central database* (CDB)
  - CDB is limited-access central record-keeping device
  - Tracks meetings between agents with access to CDB
  - History of agents with access is only available to members
- In a meeting, CDB membership observed by other members
- Agents cannot pretend to have (or not have) access

Means of exchange: Central database

The CDB entry cost κ_i is heterogeneous

- Index all agents by $i \in [0, 1]$
- Agent i's cost is $\kappa_i \geq 0$
- **A2:** κ_i is continuous, weakly increasing, weakly convex in i

$$\begin{aligned} \kappa_i &\leq \kappa_j \\ \kappa_{i+\Delta} - \kappa_i &\leq \kappa_{j+\Delta} - \kappa_j \end{aligned}$$

for $i < j$, $\Delta > 0$

Means of exchange: Central database

- CDB works like "production club"
- Contract specifies production obligations between members
- Non-binding contract: members allowed to exit at some cost

Example: A, B, C are agents with access who agree to use CDB

- If A likes what B produces, contract requires B to produce
- If A can produce what C likes, contract requires A to produce
- Money is not exchanged in either case

Timing

1. Planner sets
   - Trade quantity c_m for trades using money
   - Trade quantity c_{DB} for trades involving the CDB
   - Total money M
2. Seeing c_m, c_{DB}, agents decide whether to pay κ_i, enter CDB
3. Money is distributed
4. Economy starts

Information structure and actions

Follow mechanism design approach; formally

- Planner chooses $\{c_m, c_{DB}, M\}$ at beginning of economy
- Planner cannot observe
  - Agents in meeting,
  - Money holdings,
  - Membership to CDB
- Agents truthfully communicate meeting outcomes to planner

Information structure and actions

4 possibles cases:

Case 0: *No CDB, seller has 1 unit of money or buyer has no money. No trade.*

Case 1: *At least one agent does not have CDB access, buyer has money, seller does not.*

- **A3:** In meeting where money is used, money changes hands with probability 1
- Buyer offers money; seller produces c_m, buyer gets $u(c_m)$

Information structure and actions

Case 2: *Both agents have CDB access, buyer has no money OR seller has 1 unit of money.*

- Seller can either produce c_{DB} or 0
- If seller produces c_{DB}, buyer gets $u(c_{DB})$
- If seller refuses to produce ...
  - Both buyer and seller report (truthfully) outcome to planner
  - Seller punished by revoking CDB membership

Information structure and actions

An agent that loses CDB membership by not producing ...

- Is indistinguishable from an agent who never had access
- Can remain in the economy and rely only on money
- Cannot regain membership at a later date

Information structure and actions

Case 3: *Both agents have CDB access. Buyer has money and seller has no money.*

- Let θ be probability of using money instead of CDB
  - θ is exogenously given
- If money is used, same as Case 1
- If money is not used, same as Case 2

Information structure and actions

An agent's balance with the CDB does not matter:

- Will produce if future benefits of access outweigh immediate cost of production
- Past (sunk) actions do not affect this calculation

Value functions: no CDB access

Let $m_0 := (1 - M)/k$ and $m_1 := M/k$ be as usual; let V_0, V_1 be value functions of agents with no CDB access and 0, 1 units of money

$$V_0 = m_0(\beta V_1 - c_m) + (1 - m_0)\beta V_0$$

$$V_1 = m_1[\beta V_0 + u(c_m)] + (1 - m_1)\beta V_1$$

Can show that $V_1 > V_0$

Value functions: CDB access

Let λ be fraction of agents with access to CDB; let V_0^a, V_1^a be value functions of agents with CDB access and 0, 1 units of money

$$\begin{aligned} V_0^a &= (1-\lambda)m_1(\beta V_1^a - c_m) + \lambda m_0[u(c_{DB}) - c_{DB}] \\ &\quad + \lambda m_1[u(c_{DB}) - (1-\theta)c_{DB} + \theta(\beta V_1^a - c_m)] \\ &\quad + [1-(1-\lambda)m_1 - \lambda\theta m_1]\beta V_0^a \end{aligned}$$

$$\begin{aligned} V_1^a &= (1-\lambda)m_0[\beta V_0^a + u(c_m)] + \lambda m_1[u(c_{DB}) - c_{DB}] \\ &\quad + \lambda m_0\{(1-\theta)u(c_{DB}) + \theta[\beta V_0^a - u(c_m)] - c_{DB}\} \\ &\quad + [1-(1-\lambda)m_0 - \lambda\theta m_0]\beta V_1^a \end{aligned}$$

Incentive constraints for monetary trade

- The incentive constraints for monetary trades are

$$\begin{aligned} \beta(V_1 - V_0) &\in [c_m, u(c_m)] \\ \beta(V_1^a - V_0^a) &\in [c_m, u(c_m)] \end{aligned}$$

- **A4:** The constraints above always hold

Welfare

- Let N_0, N_1 be steady-state mass of agents with no access to CDB and carrying 0, 1 units of money
- N_0^a, N_1^a defined similarly

Lemma (1)

In the steady state, $\{N_0, N_1, N_0^a, N_1^a\}$ are given by

$$N_0 = (1-\lambda)(1-M), \quad N_1 = (1-\lambda)(1-M)$$
$$N_0^a = \lambda(1-M), \quad N_1^a = \lambda M$$

Welfare

- Want to characterize W^a, W
- In what follows, let

$$\begin{aligned} S_{DB} &:= u(c_{DB}) - c_{DB} \\ S_m &:= u(c_m) - c_m \end{aligned}$$

Welfare

Expected utility of no CDB access is $W = (1 - M)V_0 + MV_1$, or

$$W = \frac{1}{k(1-\beta)} M(1-M)S_m$$

Expected utility of CDB access is $W^a = (1 - M)V_0^a + MV_1^a$, or

$$W^a = \frac{1}{k(1-\beta)} \{\lambda[1 - \theta M(1-M)]S_{DB} + [1 - (1-\theta)\lambda]M(1-M)S_m\}$$

Welfare

Lemma (2)

Let c_{DB}, $c_m \in (0, \hat{c})$. *Then*

1. W^a *is concave in* c_{DB}, c_m *with maximum at* $u'(c_{DB}) = u'(c_m) = 1$
2. $\partial W^a / \partial \theta > 0$ *iff* $S_m > S_{DB}$
3. $\partial W^a / \partial \lambda > 0$ *iff* $[1 - \theta M(1 - M)]S_{DB} > (1 - \theta)M(1 - M)S_m$
4. W^a *is concave in* M *with maximum at* $M = 1/2$ *iff*

$$[1 - (1 - \theta)\lambda]S_m > \theta\lambda S_{DB}$$

5. W *is concave in* c_m *with maximum at* $u'(c_m) = 1$
6. W *is concave in* M *with maximum at* $M = 1/2$

2 What is the efficient allocation?

- All agents are identical except for their access cost
- Will assume that planner
  - Can observe κ
  - Can force agents to pay cost and prevent exit
- Benefit from having mass λ with access to CDB is $\lambda[W^a(\lambda) - W]$
- Lowest cost of access is $\int_0^\lambda \kappa_i \, di$

The efficient allocation

- Let SW be social welfare function:

$$\begin{aligned} SW &= \lambda W^a(\lambda) + (1-\lambda)W - \int_0^\lambda \kappa_i \, di \\ &= W + \lambda[W^a(\lambda) - W] - \int_0^\lambda \kappa_i \, di \end{aligned}$$

- From Lemma 2, efficient allocation should set $u'(c_{DB}) = u'(c_m) = 1$ and $M = 1/2$ regardless of λ

The efficient allocation

- Abstracting from access cost, marginally increasing the mass of agents with access gives benefit

$$\underbrace{[W^a(\lambda) - W]}_{private} + \underbrace{\lambda W^{a'}(\lambda)}_{network}$$

- Can show that

$$\begin{aligned} b(\lambda) &= W^a(\lambda) - W \\ &= \lambda W^{a'}(\lambda) \\ &= \lambda[\Phi(k, \beta, M, \theta, S_m, S_{DB})] \end{aligned}$$

The efficient allocation

- $b(\lambda)$ is linear function of λ

$$b(\lambda) = \lambda[\Phi(k, \beta, M, \theta, S_m, S_{DB})]$$

- Assume $\Phi(k, \beta, M, \theta, S_m, S_{DB}) > 0$
- κ is continuous, weakly increasing and weakly convex
- λ^* maximizes

$$SW = W + \lambda b(\lambda) - \int_0^\lambda \kappa_i \, di$$

3 Constrained efficient allocations

Want to describe allocations where planner

- Can choose M, c_{DB}, c_m, and can possibly influence θ
- Cannot see κ
- Cannot force agents to enter/exit

Constrained efficient allocations

- Want to obtain efficient allocations by manipulating incentive constraints
- Will focus on two different incentive constraints
  1. *Entry constraint (EC):* agent compares the expected private benefit of access to entry cost
  2. *No-exit constraint (NC):* agent compares private value of remaining in CDB with production cost

Constrained efficient allocations: entry constraint

Assume that (NC) does not bind

- Agents do not take network effects into account
  - Too few agents enter
- Agent forms belief about mass λ of agents who obtain access
- Based on belief, compares expected benefit with cost
- Enter if (EC) is satisfied:

$$W^a(\lambda) - W = b(\lambda) \geq \kappa_i$$

Entry constraint

Let c^* be such that $u'(c^*) = 1$

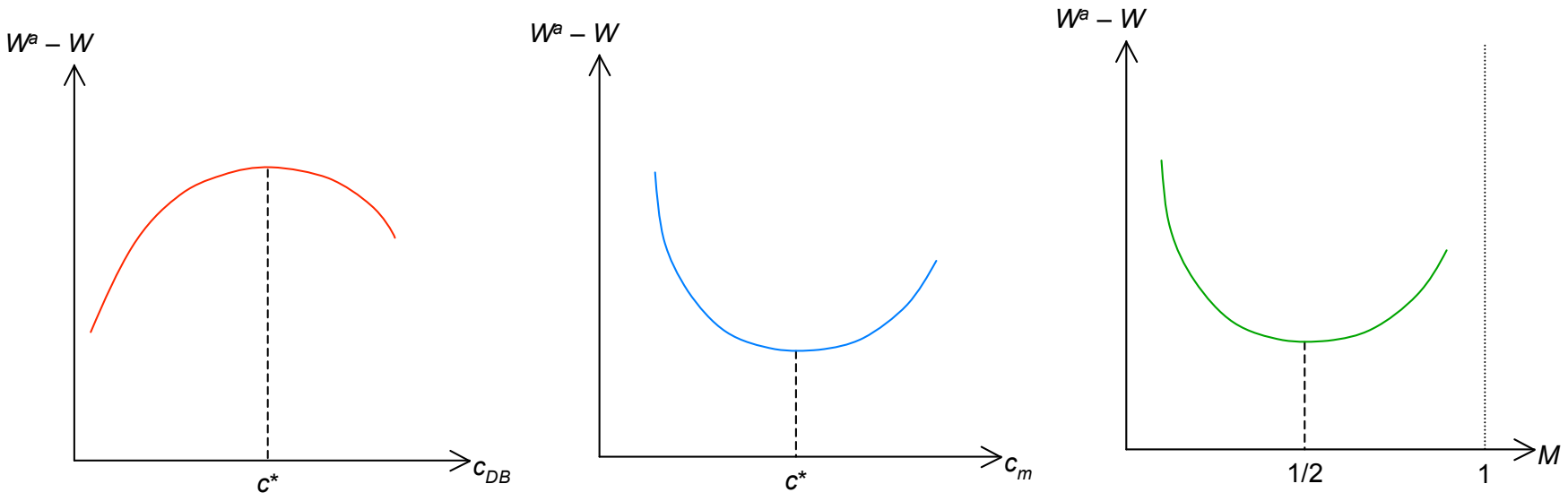
Lemma (3)

Let c_{DB}, $c_m \in (0, \hat{c})$. Then

1. $W^a - W$ *is concave in* c_{DB} *and has maximum at* $c_{DB} = c^*$
2. $W^a - W$ *is convex in* c_m *and has minimum at* $c_m = c^*$
3. $W^a - W$ *is convex in* M *and has minimum at* $M = 1/2$
4. $\partial(W^a - W)/\partial\theta > 0$ *iff* $S_m > S_{DB}$

Entry constraint

Graphically:



No-exit constraint

- (NC) is satisfied if

$$\beta(V_i^a - V_i) \geq c_{DB}$$

- Can show that $V_1^a - V_1$ is a linear function of λ

$$V_1^a - V_1 = \lambda[\Omega(k, \beta, m_0, m_1, \theta, S_m, S_{DB})]$$

- Note $\lambda \to 0$ gives $(V_1^a - V_1) \to 0$, $(W^a - W) \to 0$: it is not possible to sustain very small networks

No-exit constraint

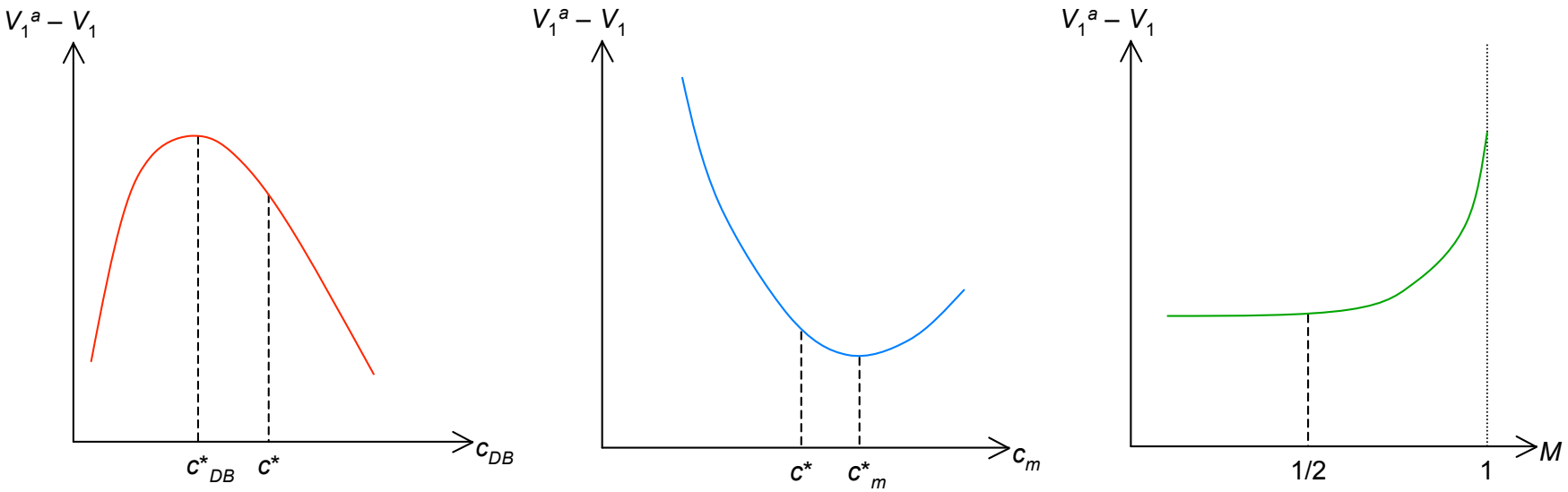
Lemma (4)

Let c_{DB}, $c_m \in (0, \hat{c})$. *Then*

1. $V_1^a - V_1 \leq V_0^a - V_0$
2. $V_1^a - V_1$ *is concave in* c_{DB} *and has maximum at* $c_{DB}^* < c^*$
3. $V_1^a - V_1$ *is convex in* c_m *and has minimum at* $c_m^* > c^*$
4. $V_1^a - V_1$ *is convex in* M*; has minimum at* $M \leq 1/2$ *and maximum at* $M = 1$
5. $\partial(V_1^a - V_1)/\partial\theta > 0$ *when* $c_m = c_{DB}$

No-exit constraint

Graphically:



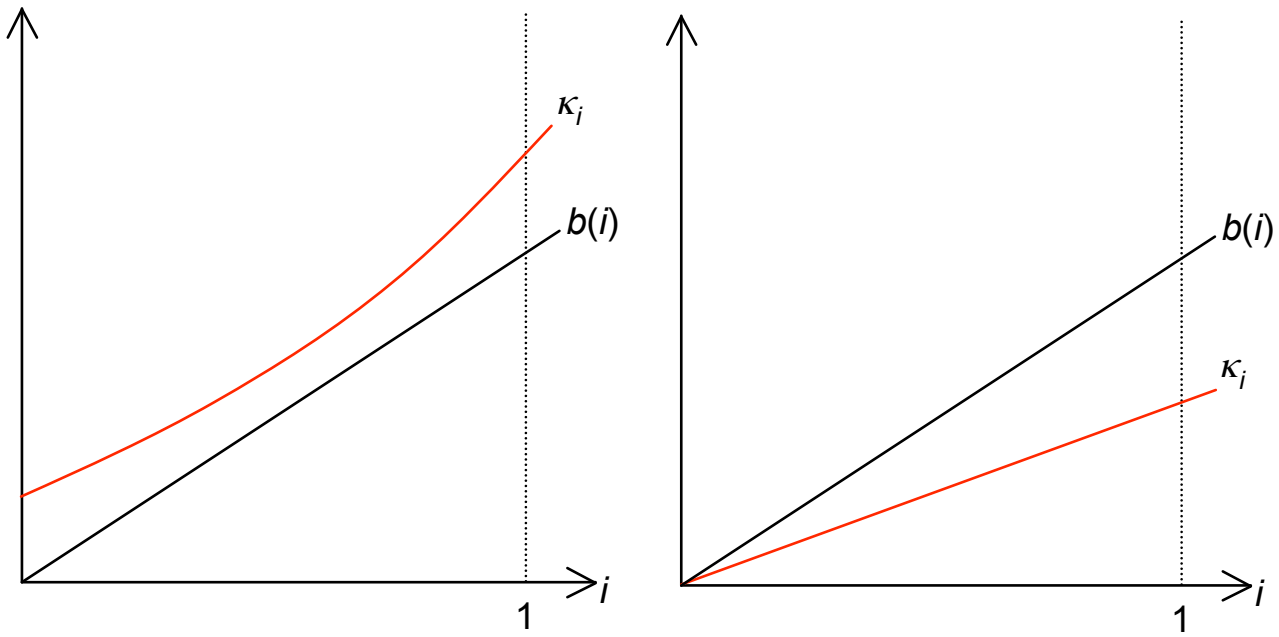
What are the constrained efficient allocations?

- In what follows, assume (NC) does not bind
- Number of equilibria will depend on graphs of $b(\lambda)$, κ
- Can identify 3 possible cases, based on the number of intersections between $b(\lambda)$ and κ

Constrained efficient allocations: zero intersections

Proposition (1-0)

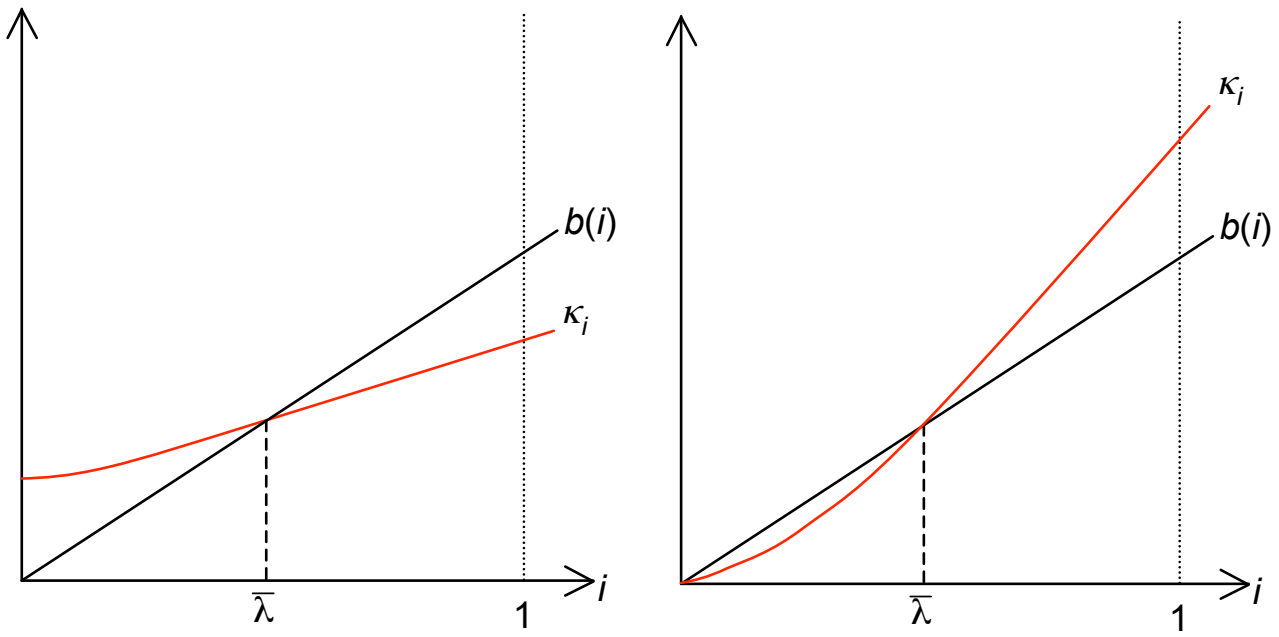
1. *If* $\kappa_i > b(i)\ \forall i$, *then* $\lambda = 0$
2. *If* $\kappa_i < b(i)\ \forall i$, *then* $\lambda \in \{0, 1\}$; $\lambda = 0$ *unstable*



Constrained efficient allocations: one intersection

Proposition (1-1)

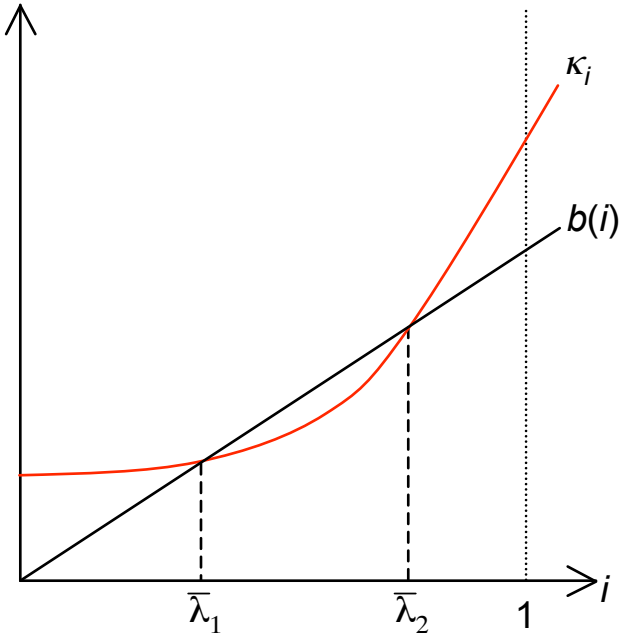
1. *If* $\kappa_i > b(i)\ \forall i \in [0, \bar{\lambda})$ *and* $\kappa_i < b(i)\ \forall i \in [\bar{\lambda}, 1]$, *then* $\lambda \in \{0, \bar{\lambda}, 1\}$; $\lambda = \bar{\lambda}$ *unstable*
2. *If* $\kappa_i < b(i)\ \forall i \in [0, \bar{\lambda})$ *and* $\kappa_i > b(i)\ \forall i \in [\bar{\lambda}, 1]$, *then* $\lambda \in \{0, \bar{\lambda}\}$; $\lambda = 0$ *unstable*



Constrained efficient allocations: two intersections

Proposition (1-2)

If $\kappa_i > b(i)$ *for* $i \in [0, \bar{\lambda}_1) \cup (\bar{\lambda}_2, 1]$ *and* $\kappa_i < b(i)$ *for* $i \in (\bar{\lambda}_1, \bar{\lambda}_2)$, *then* $\lambda \in \{0, \bar{\lambda}_1, \bar{\lambda}_2\}$

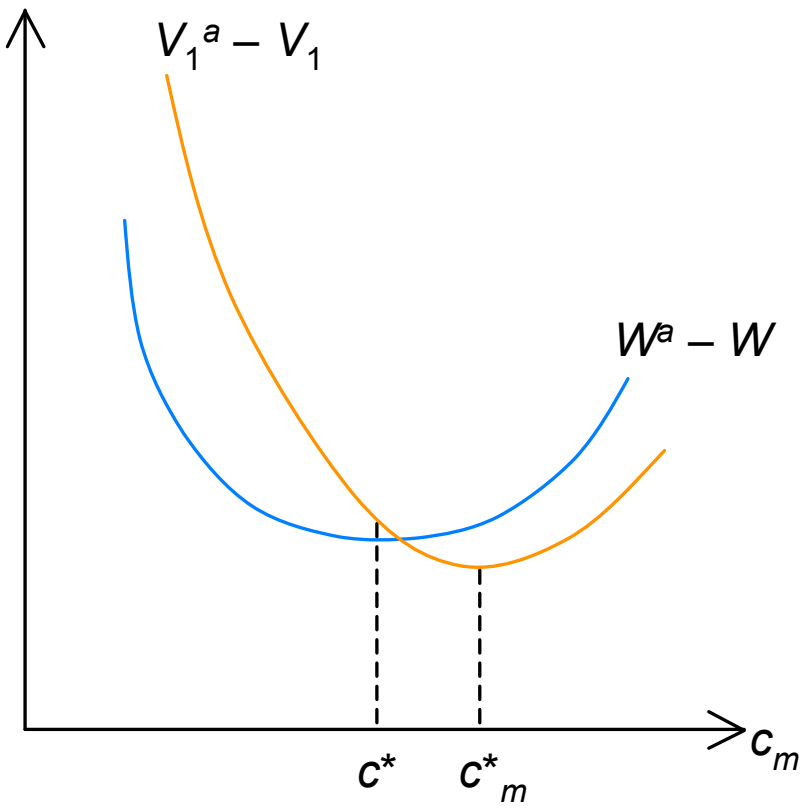


4 Relaxing the constraints

- Here we describe effect of $\{c_m, c_{DB}, M, \theta\}$ over (EC), (NC)
- Assume (NC) is not binding
- Focus on policies that relax (EC) and do not tighten (NC)

Relaxing the constraints: c_m

Can relax both constraints if c_m is decreased away from c^* or increased away from c_m^*



Relaxing the constraints: c_m

Lemma (5)

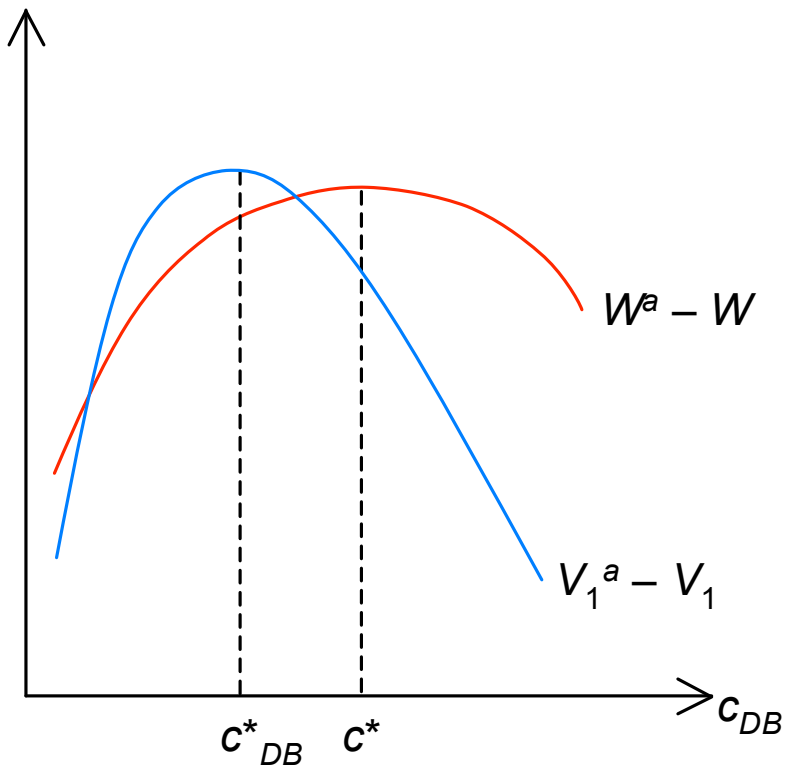
The planner always (weakly) prefers to decrease c_m below c^* than increase c_m above c_m^*

Proof (Sketch).

1. $u(c_m) - c_m$ increases welfare, relaxes (EC), tightens (NC); take two different values $\underline{c}_m < \bar{c}_m$
2. If $\underline{c}_m$, $\bar{c}_m$ give (EC) same tightness, welfare is same, planner weakly prefers $\underline{c}_m$
3. If $\underline{c}_m$, $\bar{c}_m$ give (NC) same tightness, as $u(\underline{c}_m) - \underline{c}_m > u(\bar{c}_m) - \bar{c}_m$, planner prefers $\underline{c}_m$: welfare is higher $\square$

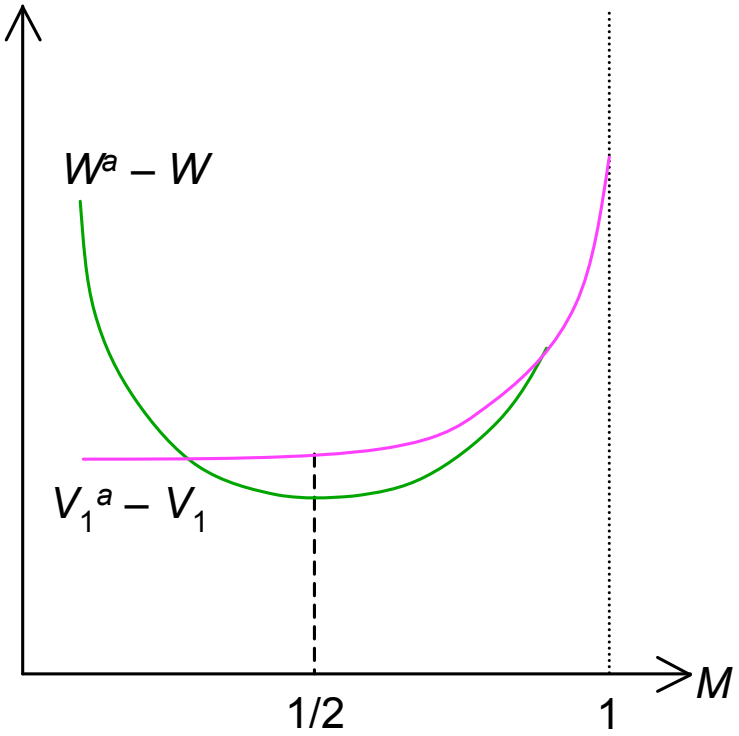
Relaxing the constraints: c_{DB}

Tighten at least one constraint if c_{DB} is increased away from c^* or decreased below c^*_{DB}



Relaxing the constraints: M

Can relax both constraints if M is increased away from $1/2$



Relaxing the constraints: No surcharge

Proposition (2)

At the unconstrained optimal values of c_m and c_{DB}, deviating from the no surcharge rule, $(c_m = c_{DB})$ must tighten at least one of the incentive constraints

Relaxing the constraints: θ

Want to consider a situation of "no surcharge" where $c_m = c_{DB}$

- If $c_m = c_{DB}$ then $S_m = S_{DB}$
- From Lemmas 3 and 4,

$$\frac{\partial(W^a - W)}{\partial\theta} = 0, \quad \frac{\partial(V_1^a - V_1)}{\partial\theta} < 0$$

- (NC) can be relaxed by decreasing θ
- How to decrease θ? (i.e. decrease chance that money is used)

Relaxing the constraints: Summary

Planner chooses $c_m = c_{DB} = c^*$ if neither constraint binds

- "Surcharge" for use of CDB means either
  - Decrease in c_{DB} (tightens (EC)), or
  - Increase in c_m (tightens (NC), unless $c_m \in (c_m^*, \hat{c})$)
- If some constraint binds, changing c_m or c_{DB} may help
  - Changing c_{DB} must tighten at least one constraint
  - Moving c_m between $[c^*, c_m^*]$ tightens at least one constraint
  - Moving c_m between $(0, c^*)$ relaxes both constraints
  - Moving c_m between $(c_m^*, \hat{c})$ relaxes both constraints

5 What policies can help achieve the efficient allocation?

- We have already seen how $\{c_m, c_{DB}, M, \theta\}$ affect constraints
- Assume (NC) is not binding
- Focus on policies that relax (EC) and do not tighten (NC):
  1. Increase M above $1/2$
  2. Reduce c_m
  3. Impose a utility tax for agents not in CDB

Limits to effectiveness of M and c_m

Proposition (3)

Suppose

$$\kappa_1 \in \left(\frac{u(c_{DB}) - c_{DB}}{k(1-\beta)}, 2\frac{u(c_{DB}) - c_{DB}}{k(1-\beta)} \right).$$

Then the efficient allocation is such that $\lambda = 1$. However it is not possible to achieve $\lambda = 1$ in the incentive constrained allocation only by changing M or c_m (or both).

Limits to effectiveness of M and c_m

Proof (Outline).

- Solving for efficient allocation, find that if $\kappa_1 < 2\frac{u(c_{DB})-c_{DB}}{k(1-\beta)}$, then $\lambda = 1$
- Highest-cost agent κ_1 will enter CDB if $[W^a(1) - W] \geq \kappa_1$
- Earlier work gives that $[W^a(1) - W] \leq \frac{1}{k(1-\beta)} S_{DB}$
- This maximum is reached if $M = 1$, $M = 0$, or if $c_m = 0$, and $(1 - \theta) = 1$
- Highest-cost agent will not join. By continuity, non-zero mass of agents will not join, so $\lambda < 1$ $\square$

Taxation

Proposition (4)

Assume the efficient allocation is such that $\lambda = 1$. *If*

$$\kappa_1 \in [W^a(1) - W, 2(W^a(1) - W)]$$

then the constrained efficient allocation is different from the efficient allocation. The efficient allocation can be achieved by setting a high enough tax for agents who do not access the CDB.

Taxation

Proof (Outline).

- If $\kappa_1 \geq [W^a(1) - W]$ some agents will not enter; yet $\lambda = 1$ socially optimal if $\kappa_1 \in [W^a(1) - W, 2(W^a(1) - W)]$
- Impose tax $T \geq \kappa_1$ on any agent who *does not* enter CDB
- Now individually rational for all agents to join
- Note: Since all agents join, no tax is paid $\square$

If $\lambda < 1$ is optimal ...

- Want to maximize

$$SW = \lambda W^a + (1-\lambda)(W - T) - \int_0^\lambda \kappa_i di$$

- Will compare three policies:
  1. $T = \bar{T} > 0$, $M = \frac{1}{2}$, $c_m = c^*$
  2. $T = 0$, $M = \bar{M} \neq \frac{1}{2}$, $c_m = c^*$
  3. $T = 0$, $M = \frac{1}{2}$, $c_m = \bar{c}_m \neq c^*$
- Assume $c_{DB} = c^*$
- $W(T_0, M_0, c_0)$: welfare with taxes T_0, money supply M_0, trade quantity for money exchanges c_0

Welfare implications of policy

Proposition (5)

$W(\bar{T}, \frac{1}{2}, c^*) - \bar{T} \geq W(0, \bar{M}, c^*) \geq W(0, \frac{1}{2}, \bar{c}_m)$

Proof (Intuition).

Size of wedge needed to get marginal agent to enter is same for all policies. Increasing M or decreasing c_m decreases utility of money transaction for *both* CDB and non-CDB agents. With tax, only agents who do not access CDB are affected. □

Remark

If (NC) binds, then there may be a role for increasing M

6 What if no unit upper bound?

Use Berentsen approach ...

- Agents can hold N units of money
- Indivisible goods, indivisible money
- Let m_n denote the fraction of the population of n units of money, $\sum_{i=1}^{N} nm_n = M$
- Let p_b denote the probability of meeting a money-holding buyer, $p_b = \frac{1-m_0}{k}$
- Let p_s denote the probability of meeting a seller who can accept money, $p_s = \frac{1-m_N}{k}$

Value functions: no-access

Under no access,

$$\begin{aligned} V_0 &= p_b(\beta V_1 - c_m) + (1 - p_b)\beta V_0 \\ V_n &= p_b(\beta V_{n+1} - c_m) + p_s(\beta V_{n-1} + u(c_m)) + (1 - p_b - p_s)\beta V_n \\ V_N &= p_s(\beta V_{N-1} + u(c_m)) + (1 - p_s)\beta V_N \end{aligned}$$

Expected utility is $W = \sum_{n=0}^{N} m_n V_n$, or

$$W = m_0 V_0 + m_N V_N + \sum_{n=1}^{N-1} m_n V_n$$

Value functions: CDB access

Under access,

$$
\begin{aligned}
V_n^a \quad = \quad & (1-\lambda)[p_b(\beta V_{n+1}^a - c_m) + p_s(\beta V_{n-1}^a + u(c_m))] \\
& +\lambda\left(\frac{m_N}{k}u(c_{DB}) - \frac{m_0}{k}c_{DB}\right) \\
& +\lambda p_b\left[\theta(\beta V_{n+1}^a - c_m) - (1-\theta)c_{DB}\right] \\
& +\lambda p_s\left[\theta(\beta V_{n-1}^a + u(c_m)) + (1-\theta)u(c_{DB})\right] \\
& +[1-(1-\lambda)(p_b+p_s) - \lambda\theta(p_b+p_s)]\beta V_n^a
\end{aligned}
$$

Expected utility is $W^a = \sum_{n=0}^{N} m_n V_n^a$, or

$$
W^a = m_0 V_0^a + m_N V_N^a + \sum_{n=1}^{N-1} m_n V_n^a
$$

No unit upper bound

Impose $m_0 = m_n = m_N \Rightarrow p_b = p_s$

- W^a, W, $W^a - W$ still concave (convex) in c_{DB} (c_m) with maximum at $u'(c_{DB}) = u'(c_m) = 1$
- $V_n^a - V_n$ still still concave (convex) in c_{DB} (c_m)
- W reaches maximum at $M = \frac{n}{2}$
- Concavity and convexity in M ... maybe
- Taxation as optimal policy still true

7 Comparing with Cavalcanti-Wallace

- Compare with "no-gift" allocations
- Assume both economies have same set-up but CW agents pay cost to make histories public information, λ^p
- In a meeting:
  - If two public-history agents, production always occurs
  - If two private-history agents, production occurs if buyer has money, seller does not
  - If public-private, production occurs if private agent wants to consume and has money, or can produce and does not have money
- Assume $M = \frac{1}{2}$, exogenous; $c_m = c_{DB} = c$, (NC) not binding

Comparing with Cavalcanti-Wallace

Proposition (6)

If $\lambda^a = \lambda^p = \lambda$, then $SW_{CW} \geq SW_{CDB}$ for all λ

Proof (Outline).

Algebra gives

$$\begin{aligned} SW_{CW} &= \frac{(1+\lambda^p)^2}{4}\frac{u(c)-c}{k(1-\beta)} - \int_0^{\lambda^p} \kappa_i di \\ SW_{CDB} &= \frac{1+(3\lambda^a)^2}{4}\frac{u(c)-c}{k(1-\beta)} - \int_0^{\lambda^a} \kappa_i di \end{aligned}$$

Intuition:

- More non-money trades in CW
- Can support small networks in CW

□