Financial Intermediaries and Markets

Franklin Allen and Douglas Gale, Econometrica, 2004

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Financial Crisis: Is it bad?

- Common view: Financial crises are inefficient.
- Reason: Coordination failure causes attack
- Policy implication: governments must intervene.

  Example: Diamond-Dybvig(83), Obstfeld(96) and many others.

Financial Crisis: Is it bad? - Continued

- This Paper: Efficinecy depends on the market and contract structure.

	Complete Markets	Incomplete Markets
Incomplete Contracts	incentive-efficient	not efficient
Complete Contracts	constrained-efficient	not efficient

- When markets are incomplete, regulation in the form of a lower bound on liquidity, improves upon the market allocation.

Economy

- Time: $t = 0, 1, 2$.
- State of the economy, $\eta \in H$; realized in $t = 1$, according to distribution $\nu(\eta)$.
- Two "assets": Long and Short.
  1. Short/Storage: 1 unit invested in $t = 0, 1$ returns 1 unit in $t + 1$.
  2. Long: 1 unit invested in $t = 0$ returns $R(\eta) > 1$ in period 2, if η is realized; cannot be liquidated.

Investors

- n different ex-ante type; indexed by i.
- Each type has a mass of μ_i;$\sum_i \mu_i = 1$.
- at date 1, each ex-ante type i receives an idiosyncratic shock $\theta_i \in \Theta_i$.
- Preferences: Ex-post type θ_i values consumption in $t = 1, 2$ according to $u_i(c_1, c_2, ; \theta_i)$
- $\Pr((i, \theta_i)|\eta) = \lambda_i(\theta_i, \eta)$.

Investors - Continued

- Endowments: 1 unit at date 0, 0 units at $t = 1, 2$.
- Investors have access to storage technology at date 1. No access to any saving technology at date 0.
- Financial intermediary attached to each ex-ante type.

Markets

- Date 0: Markets for date-1 Arrow securities open; price $q(\eta)$
- Date 1: Spot and Forward markets in terms of date 1 good open; price $p(\eta) = (1, p_2(\eta))$.
  This makes markets complete.
- At date 0, agent i deposit endowment in exchange for a contract: $\{x_i(\theta_i, \eta) \in \mathbb{R}^2_+\}$.

Contracts

- *Revelation Principle* holds; Direct mechanisms.
- Feasible consumptions given the contract:

$$C(x_i(\theta_i,\eta)) = \left\{(c_1,c_2); c_1 \leq x_{i1}(\theta_i,\eta), \sum_{t=1}^{2} c_t \leq \sum_{t=1}^{2} x_{it}(\theta_i,\eta)\right\}$$

- indirect utility function:

$$u_i^*(x_i(\theta_i,\eta),\theta_i) = sup_{c\in C(x_i(\theta_i,\eta))} u_i(c,\theta_i)$$

Contracts - Continued

- Incentive Compatibility:

$$u_i(x_i(\theta_i, \eta), \theta_i) \geq u_i^*(x_i(\hat{\theta}_i, \eta), \theta_i), \forall \theta_i, \hat{\theta}_i \in \Theta_i, \forall \eta \in H$$

same as hidden information and saving á la Cole-Kocherlakota(2001).
X_i: set of incentive compatible mechanisms.

Intermediary's Problem

- Intermediary i takes μ_i units of endowment and invests y_i units in the short asset and the rest in the long asset.
- Given $p, q, \bar{u}$, intermediary solves this problem:

$$\max_{x_i, y_i} \sum_{\eta} q(\eta) p(\eta) \cdot (y_i, (\mu_i - y_i) R(\eta)) - \sum_{\eta} q(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) p(\eta) \cdot x_i(\theta_i, \eta)$$

$$\text{s.t.} \quad x_i \in \hat{X}_i,$$
$$\sum_{\eta} \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i(x_i(\theta_i, \eta), \theta_i) \geq \bar{u}$$

Equilibrium

Definition

A *Pure Intermediated Equilibrium* is $((x_i^*, y_i^*, \bar{u}_i)_{i=1,\ldots,n}, (p^*, q^*))$ such that

1. $\forall i = 1, \ldots, n$, given $(p^*, q^*, \bar{u}_i)$, (x_i^*, y_i^*) solves intermediary i's problem.
2. (x_i^*, y_i^*) is *attainable*:

$$\sum_i \sum_{\theta_i} \lambda_i(\theta_i, \eta) x_{i1}(\theta_i, \eta) \leq \sum_i y_i, \quad \forall \eta$$

$$\sum_i \sum_{\theta_i} \lambda_i(\theta_i, \eta) \left(x_{i1}(\theta_i, \eta) + x_{i2}(\theta_i, \eta) \right) = \sum_i y_i + (\mu_i - y_i) R(\eta), \quad \forall \eta$$

3. $\forall i, \bar{u}_i = \sum_\eta \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i(x_i^*(\theta_i, \eta), \theta_i)$.
4. Profits are zero; Perfect Competition + Free Entry.

Non-convexities and Mixed Equilibrium

- Notice: X_i is not convex in general.
- Cannot apply Kakutani's fixed point theorem to the excess demand to get existence of equilibrium.
- Prescott and Townsend's idea: Allow for lotteries; allowing only for finite sets of allocation.
- Here: More restricted notion of lotteries;

Mixed Equilibrium

- Mixed allocation: divide the population into J groups with size ρ_j, $j = 1, .., J$. The frequency of each ex-ante type in each group is μ_i.
- A type i in group j is matched with an intermediary; receives a contract (x_i^j, y_i^j).
- Note: Mixed equilibrium implies, each ex-ante type i must be indifferent between (x_j^i, y_j^i), $\forall j$.
- A mixed allocation is *attainable* if $\sum_j \rho^j (x_j^i, y_i^j)$ is attainable. Notice possibility of trade among groups of the same type.

Existence of Equilibrium

Assumption (Non-empty Interior)

For any ex-ante type $i = 1, .., n$, for any consumption bundle $x_i \in X_i$ and price system $p \in P$, and for any $\epsilon > 0$, either

$$\sum_{\eta} p(\eta) \cdot \left(\sum_{\theta_i} \lambda_i(\theta_i, \eta) x_i(\theta_i, \eta) \right) = 0$$

or

$$\exists x_i' \in \mathbb{B}_\epsilon(x_i) :$$

$$\sum_{\eta} p(\eta) \cdot \left(\sum_{\theta_i} \lambda_i(\theta_i, \eta)(x_i'(\theta_i, \eta) - x_i(\theta_i, \eta)) \right) < 0$$

Existence - Continued

Theorem (1)

Under nonempty interior assumption, there exists a mixed intermediated equilibrium.

- Idea: show that intermediary's problem is equivalent to the following dual problem:

$$
\begin{aligned}
\max_{x_i, y_i} \quad & \sum_{\eta} \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i(x_i(\theta_i, \eta), \theta_i) \\
\text{s.t.} \quad & x_i \in X_i \\
& \sum_{\eta} q(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) p(\eta) \cdot x_i(\theta_i, \eta) \leq \\
& \sum_{\eta} q(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) p(\eta) \cdot (y_i, (\mu_i - y_i) R(\eta))
\end{aligned}
$$

Existence - Continued

- This is similar to a demand problem. We can define aggregate excess demand correspondence.
- We can convexify it using Carathéodory theorem and apply Kakutani's fixed point theorem to show equilibrium exists.
- We will restrict attention to pure intermediated equilibria.

Efficiency

Definition

An attainable allocation (x, y) is *incentive-efficient* if there does not exist an attainable mixed allocation (ρ^j, x_i^j, y_i^j) such that:

$$\sum_{\eta} \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i(x_i^j(\theta_i, \eta), \theta_i) \geq \sum_{\eta} \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i(x_i(\theta_i, \eta), \theta_i)$$

for all (i, j) with a strict inequality for at least one (i, j) and $x_i^j \in X_i$.

Efficiency - Continued

Assumption (Local Nonsatiation)

for any ex-ante type i, for any mechanism $x_i \in X_i$, and for any $\epsilon > 0$, there exists a mechanism $x_i' \in X_i \cap \mathbb{B}_\epsilon(x_i)$ such that

$$\sum_\eta \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i(x_i'(\theta_i, \eta), \theta_i) \geq \sum_\eta \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i(x_i(\theta_i, \eta), \theta_i)$$

- Equivalent to the usual local nonsatiation property only stronger.

A First Welfare Theorem

Theorem

If $(p, q, x, y, \bar{u})$ is an intermediated equilibrium and Local Nonsatiation is satisfied, then the allocation (x, y) is incentive-efficient.

First Welfare Theorem - Continued

- Idea of proof: the same as GE FWT;
- If there exists a better allocation, it will cost more in total and contradicts feasibility.

Ex. 1: Basic Setup

- Single ex ante type
- Diamond-Dybvig preferences
- Two ex post types: early ($\theta = 1$), late ($\theta = 2$)

$$u(c_1, c_2, \theta) = \begin{cases} \log c_1, & \theta = 1 \\ \log c_2, & \theta = 2 \end{cases}$$

- $H = \{1, 2\}, \nu(1) = \nu(2) = 1/2$
- $P(\theta = 1|\eta) = 1/2, \eta = 1, 2$
- Contingent returns to long asset:

$$R(1) = r; \;\; R(2) = 2.$$

Solutions

- In general, solution depends on r
- Approach:
  - solve for efficient allocation and find the right way to decentralize using no-arbitrage conditions.
  - Constraints to check: liquidity constraint; incentive constraint.
  - Changing interest rate r changes the status of IC and LC.

Ex. 1 - Intuition

- Incentive Constraint : $c_2(\eta) \geq c_1(\eta)$.

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- Example 1B: stochastic interest rate not too high; Hard to give date-2 consumption; IC binds in low state; Need extra liquidity in the low state.
- Example 1C: stochastic interest rate too low; IC binds in both states; Slack liquidity constraint in both states;

Ex. 1 - Results

- Allocations

Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$	$E[U]$
1A	1.5	.5	(1,1.5)	(1,2)	.275
1B	.5	.64	(.82,.82)	(1.28,1.44)	.054
1C	.3	.79	(.85,.85)	(1.21,1.21)	.016

Example 4

Ex. 1 - Results

- Allocations

Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$	$E[U]$
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Example 4

- Prices

Example	$(q(1), p_2(1))$	$(q(2), p_2(2))$
1A	(.5,.67)	(.5,.5)
1B	(.61,1)	(.39,.89)
1C	(.59,1)	(.41,1)

Ex. 2: Basic Setup

- Two ex ante types with D-D preferences
  - A: risk averse
  - N: risk neutral
- unit measure of each ex ante type
- period preferences:

$$u_A(c) = \log(c); \;\; u_N(c) = c.$$

- $P(\theta_i = 1|\eta) = 1/2, \eta = 1, 2, i = A, N$

Ex. 2 - Results

- Allocations

Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$
2A	.5	.5	(1,1.25)	(1,1.25)
2N	.5	0	(.82,.82)	(1.28,1.44)

Ex. 2 - Results

- Allocations

Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$
2A	.5	.5	(1,1.25)	(1,1.25)
2N	.5	0	(.82,.82)	(1.28,1.44)

- Prices

Example	$(q(1), p_2(1))$	$(q(2), p_2(2))$
2	(.5,.8)	(.5,.8)

Ex. 3: Basic Setup

- like Ex. 1
- $P(\theta = 1|1) = 0.4, P(\theta = 1|2) = 0.6$
- Returns to long asset noncontingent:

$$R(1) = R(2) = r.$$

Ex. 3: Basic Setup

- like Ex. 1
- $P(\theta = 1|1) = 0.4, P(\theta = 1|2) = 0.6$
- Returns to long asset noncontingent:

$$R(1) = R(2) = r.$$

- 1 ex-ante type implies aggregate shock.

Assumptions

- General ad-hoc incomplete contracts
- Maintain complete markets
- Exogenous bankruptcy institution

Bankruptcy/Default Institution

- If Intermediary defaults at $t = 1$
  - assets liquidated
  - proceeds distributed to investors
  - note: CM $\Rightarrow$ arrow securities default-free
- Institution of bankruptcy is exogenous.
- A general bankruptcy code : $w_i(\eta) = F_i(y_i, \eta)$.
- In the examples:

$$w_i(\eta) = y_i + (\mu_i - y_i)R(\eta)$$

Intermediary's Choice Set

- $x_i \in X_i^I$: incomplete contracts when no default
- $\bar{x}_i(\theta_i, \eta) = (w_i(\eta), 0)$: default contract
  - $w_i(\eta)$: liquidated value of assets
- $B_i : H \to \{0, 1\}$, $B_i(\eta) = 1 \Rightarrow$ default
- $y_i \geq 0$.

Intermediary's Choice Set - continued

- Effective Contract, $\hat{x}_i$:

$$\hat{x}_i(\theta_i, \eta) = (1 - B(\eta))x_i(\theta_i, \eta) + B(\eta)\bar{x}_i(\theta_i, \eta)$$

- Incentive Compatibility:

$$u_i^*(\hat{x}_i(\theta_i, \eta), \theta_i) \geq u_i^*(\hat{x}_i(\hat{\theta}_i, \eta), \theta_i), \forall\ \theta_i, \hat{\theta}_i \in \Theta_i.$$

- $\hat{X}_i$: Incentive Compatiable effective contracts

Example: Deposit Contracts

- Restrict intermediaries to noncontingent contracts:

$$x_i \in X_i^I \Leftrightarrow x_i(\theta_i, \eta) = x_i(\theta_i, \eta') \ \forall \ \eta, \eta' \in H$$

- Assume Diamond-Dybvig preferences:

$$x_i(\theta_i, \eta) = \begin{cases} (c_1, 0) & \text{if } \theta_i = 0, \\ (0, c_2) & \text{if } \theta_i = 1. \end{cases}$$

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$$x_i \in X_i^I \Leftrightarrow x_i(\theta_i, \eta) = x_i(\theta_i, \eta') \ \forall \ \eta, \eta' \in H$$

- Assume Diamond-Dybvig preferences:

$$x_i(\theta_i, \eta) = \begin{cases} (c_1, 0) & \text{if } \theta_i = 0, \\ (0, c_2) & \text{if } \theta_i = 1. \end{cases}$$

- Default loosens the constraint set:

$$\hat{x}_i(\theta_i, \eta) = \begin{cases} (c_1, 0) & \text{if } \theta_i = 0, B_i(\eta) = 0, \\ (0, c_2) & \text{if } \theta_i = 1, B_i(\eta) = 0, \\ (w_i(\eta), 0) & \text{if } B_i(\eta) = 1 \end{cases}$$

Intermediary's Problem

- Given $p, q, \bar{u}, F_i,$, intermediary i chooses $B_i, \hat{x}_i, y_i$ to solve

$$\max_{B_i,\hat{x}_i,y_i} \sum_\eta q(\eta)p(\eta) \cdot (y_i, (\mu_i - y_i)R(\eta))$$

$$-\sum_\eta q(\eta)\left(B_i(\eta)w_i(\eta) + (1 - B_i(\eta))\sum_{\theta_i} \lambda_i(\theta_i,\eta)p(\eta) \cdot x_i(\theta_i,\eta)\right)$$

$$\begin{aligned} \text{s.t.} \quad & \hat{x}_i \in \hat{X}_i \\ & \sum_\eta \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i,\eta) u_i^*(\hat{x}_i(\theta_i,\eta),\theta_i) \geq \bar{u} \\ & w_i(\eta) = F_i(y_i,\eta) \end{aligned}$$

Feasibility

- Given an intermediary's allocation, $(B_i, \hat{x}_i, y_i)$, define θ_i's optimal consumption plan in case of default, b_i:

$$b_i(\theta_i, \eta) = \arg\sup \{u_i(c, \theta_i) | \quad c_1 \leq w_i(\eta) \\ c_1 + c_2 \leq w_i(\eta)\}$$

Feasibility

Definition

An allocation $(B, \hat{x}, y)$ is *attainable* if and only if

$$\sum_i \sum_{\theta_i} \lambda_i(\theta_i, \eta) \left(B_i(\eta) b_{i1}(\theta_i, \eta) + (1 - B_i(\eta)) x_{i1}(\theta_i, \eta)\right) \leq \sum_i y_i, \ \forall \eta$$

$$\sum_i \sum_{\theta_i} \lambda_i(\theta_i, \eta)(B_i(\eta)(b_{i1}(\theta_i, \eta) + b_{i2}(\theta_i, \eta))$$
$$+(1 - B_i(\eta)(x_{i1}(\theta_i, \eta) + (x_{i2}(\theta_i, \eta))) = \sum_i y_i + (\mu_i - y_i) R(\eta), \forall \eta$$

Equlibrium

Definition

A *Pure Intermediated Equilibrium with Incomplete Contracts* is $((B_i^*, \hat{x}_i^*, y_i^*, \bar{u}_i)_{i=1,\ldots,n}, (p^*, q^*))$ such that

1. $\forall i = 1, \ldots, n$, given $(p^*, q^*, \bar{u}_i, F_i)$, $(B_i^*, \hat{x}_i^*, y_i^*)$ solves intermediary i's problem.
2. $(\hat{x}_i^*, y_i^*)$ is *attainable*:
3. $\forall i, \bar{u}_i = \sum_\eta \nu(\eta) \sum_{\theta_i} \lambda_i(\theta_i, \eta) u_i^*(\hat{x}_i^*(\theta_i, \eta), \theta_i)$.
4. Profits are zero; Perfect Competition + Free

Efficiency

- $\hat{X}_i \subset X_i$.
- Constrained-Efficiency: Same as incentive-efficient only restrict allocations to $\hat{X}_i$.
- Analogues of Assumption 2, Theorem 2:

Theorem (4)

If $(B, \hat{x}, y, \bar{u}, p, q)$ is an intermediated equilibrium with incomplete contracts and Assumption 2' is satisfied, then $(B, \hat{x}, y)$ is constrained-efficient.

Mixed Equilibrium

- Notion of mixed-equilibrium can be defined similar to the complete contract case.

- Notice: It seems that we cannot restrict attention to pure equilibria with default.
  Suppose there is a single ex-ante type and constrained-efficiency involves default in some state. Pure equilibrium implies that the single bank should default in that state.
  This drives p_2 in that state to 0 and therefore there is room for entry of a solvent bank.

Ex. 4 - Equilibrium with Default

- Same as example 1; Allow for default and only deposit contracts
- 4A: If r high enough, can implement the unconstrained Pareto Efficient.

  Example 1

  Prices are not determined completely; deposit contracts prevent arbitrage between states at date 1.

Ex. 4 - Continued

- Prices:
  1. CM $\Rightarrow q(\eta)p_2(\eta) = \lambda(2,\eta)u'(c_2(\eta))$,
  2. No-arbitrage: $q(1) + q(2) = 1$.
  3. No-arbitrage: $p_2(\eta) \leq 1$.

Ex. 4 - Continued

- Prices:
  1. CM $\Rightarrow q(\eta)p_2(\eta) = \lambda(2,\eta)u'(c_2(\eta))$,
  2. No-arbitrage: $q(1) + q(2) = 1$.
  3. No-arbitrage: $p_2(\eta) \leq 1$.
- This gives a range on $q(1)$:
  $q(1) \in [\lambda(2,1)u'(c_2(1)), 1 - \lambda(2,2)u'(c_2(2))]$

  In this case: $q(1) \in [.33, .75]$

Ex. 4 - Continued

- 4B: If $r < 1$, cannot decentralize the incentive-efficient allocation with deposit contracts.
  Example 1
- Need default to make consumption state-contingent,
- In a pure equilibrium, every one will default in state 1 and want to liquidate. This implies $p_2(1) = 0$.
  If $p_2(1) = 0$, a bank can enter and buy all the long assets at date 2 for zero cost and sell date 1 good and make huge profit.
- No pure equilibrium with all banks defaulting in state 1 exists; mixed equilibrium.

Ex.4 - Continued

- To find equilibrium, we can solve for constrained Pareto efficient allocation with default.
- 4B: $0.9 \leq r < 1$: it turns out that in this case deposit contracts give higher utility than default.
- Need to find prices to decentralize the constrained efficient.

Ex.4 - Continued

- Prices:
  1. CM $\Rightarrow q(\eta)p_2(\eta) = \lambda(2,\eta)u'(c_2(\eta))$.
  2. No-Arbitrage : $q(1) + q(2) = 1$.

- To prevent banks from default, we need Cost(Default)>Benefit(Default).
- Benefit to banks from default:

$$u(\eta)_{\text{default}} - q(\eta)w_i(\eta)$$

- Cost of default:

$$u(\eta)_{\text{solvent}} - \sum_{\theta_i} \lambda_i(\theta_i,\eta)p(\eta)\cdot x_i(\theta_i,\eta)$$

Ex. 4 - Continued

- Doing the above for $\eta = 1, 2$, gives an upper bound and lower bound on $q(1)$.
- Results:
  Allocations

Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$
4B	.95	.49	(.97,.97)	(.97,2.05)

Ex. 4 - Continued

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Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$
4B	.95	.49	(.97,.97)	(.97,2.05)

Prices:

Example	$(q(1), p_2(1))$	$(q(2), p_2(2))$
4B	(.55-.77,.70-.98)	(.45-.33,.53-.73)

Ex. 4 - Continued

- As r goes down, the two bounds above converge and for $r < .9$, the range is empty. So we must have a mixed banking equilibrium. A fraction $\rho < 1$ of the banks remain solvent and the rest declare bankruptcy.
- As $r \to 0$, $\rho \to 0$.

Ex. 4 - Continued

- For $r = .5$:
  Allocations

Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$
4C S	.5	.67	(.95,.95)	(.95,1.41)
4C B		0	(.77,.77)	(1.41,1.41)

Ex. 4 - Continued

- For $r = .5$:
  Allocations

Example	r	y	$(c_1(1), c_2(1))$	$(c_1(2), c_2(2))$
4C S	.5	.67	(.95,.95)	(.95,1.41)
4C B		0	(.77,.77)	(1.41,1.41)

Prices:

Example	$(q(1), p_2(1))$	$(q(2), p_2(2))$
4C	(.65,.90)	(.35,1)